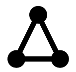
10 Problems for Partitions of Triangle-free Graphs

Balogh, Clemen, Lidický, 2022

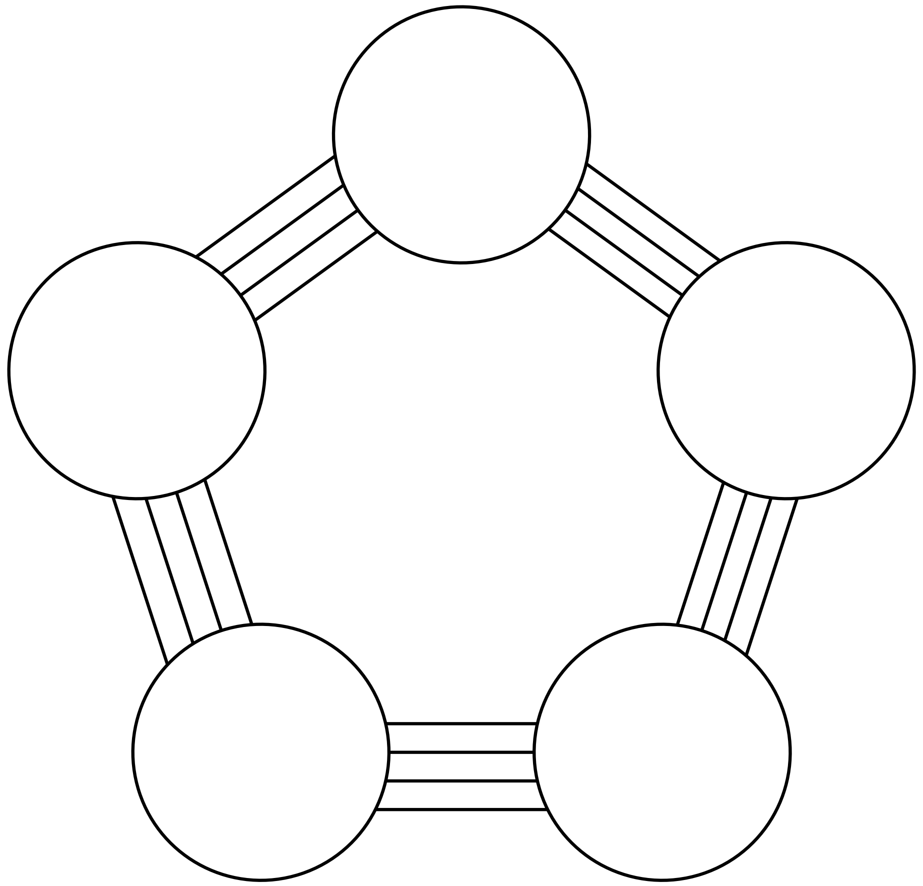
Ref. Ignacy Buczek

Sparse half

Erdős, 1976

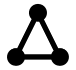
-free graphs

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?



Sparse half

Erdős, 1976

-free graphs

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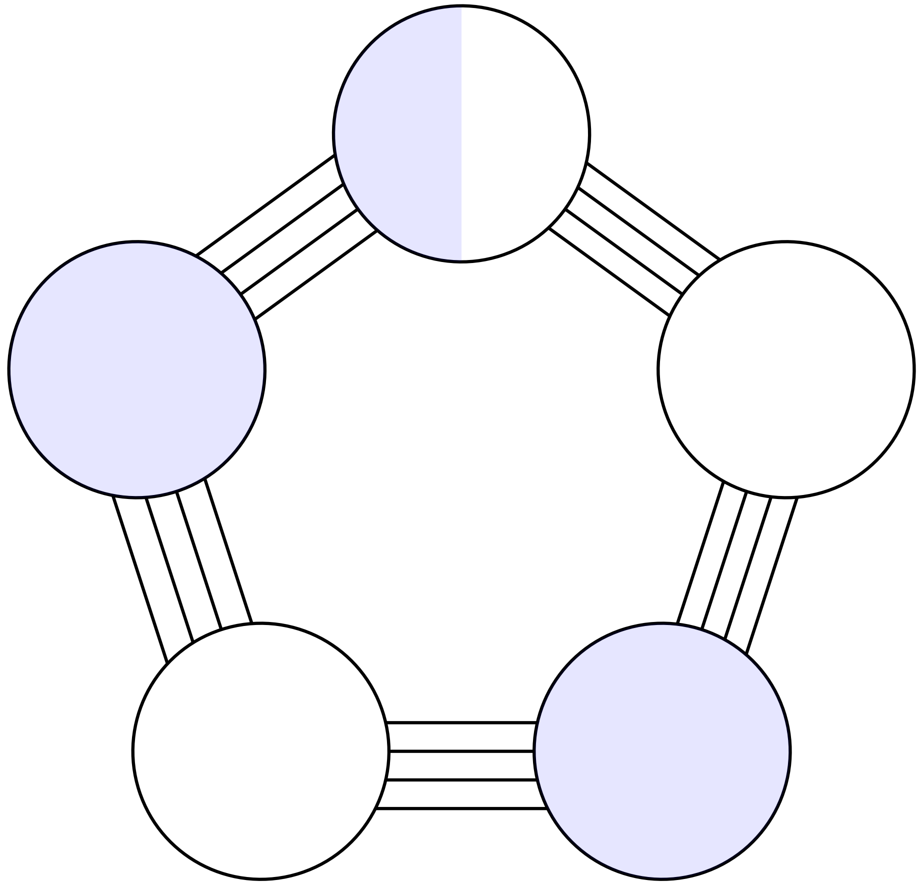
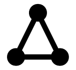


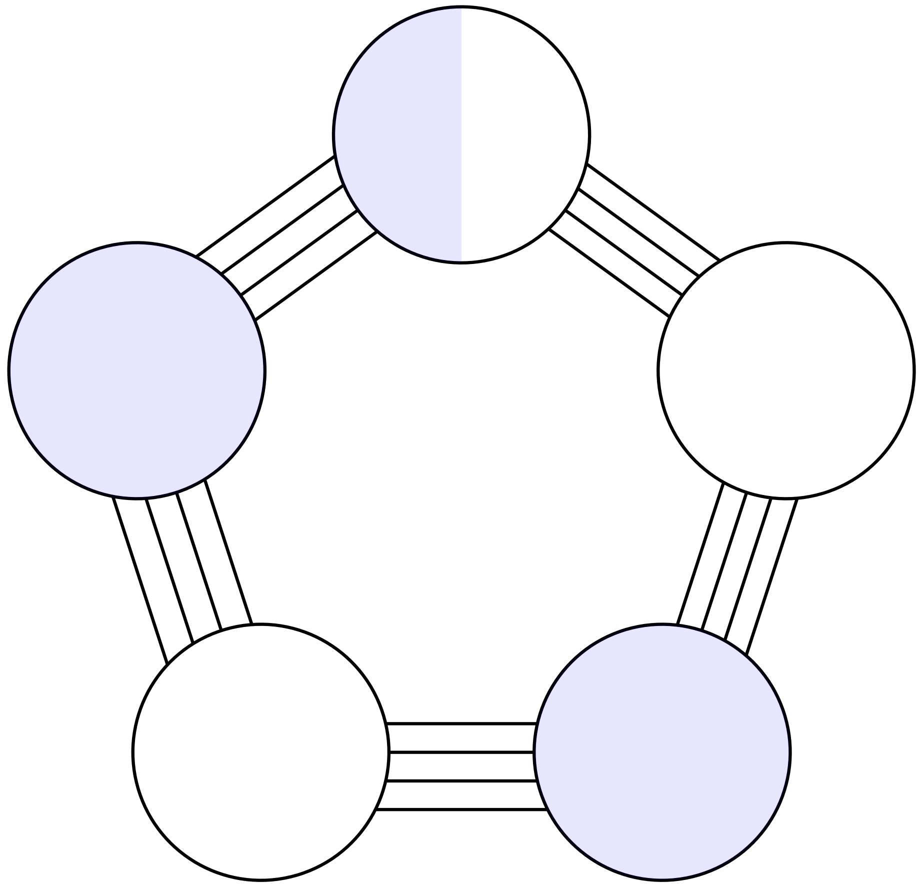
Sparse half

Erdős, 1976

-free graphs

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\frac{n}{2} = \lfloor \alpha n \rfloor$ $\qquad$ $\frac{n^2}{50} = \lfloor \beta n^2 \rfloor$



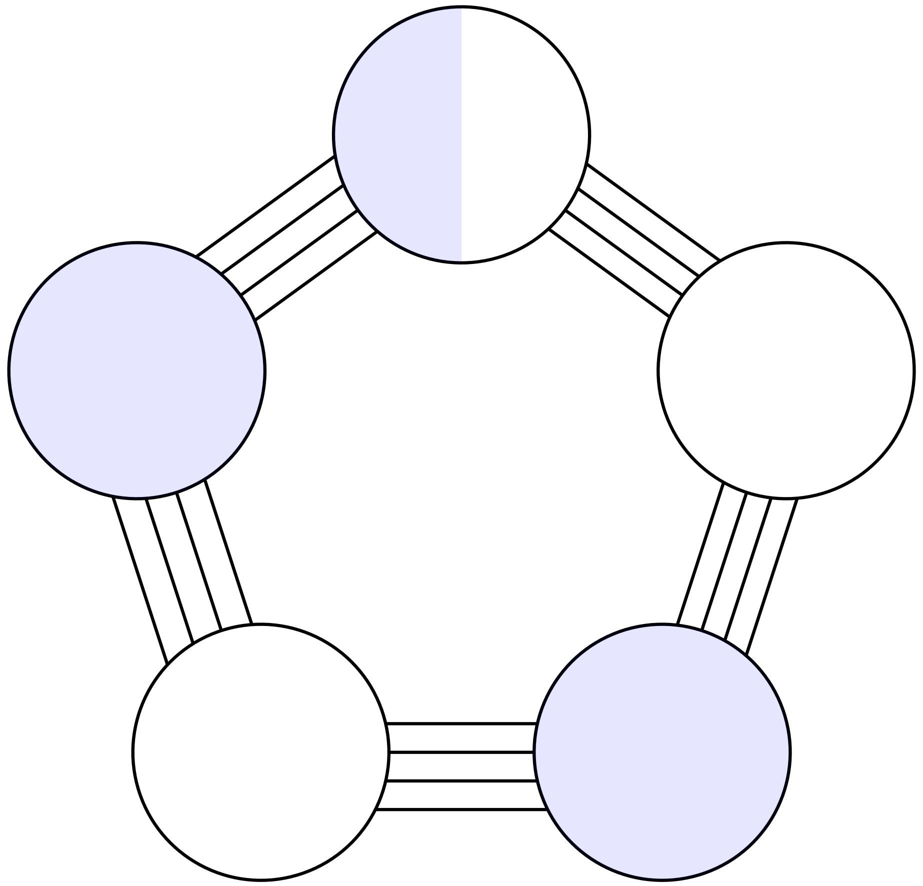
Sparse half

Erdős, 1976

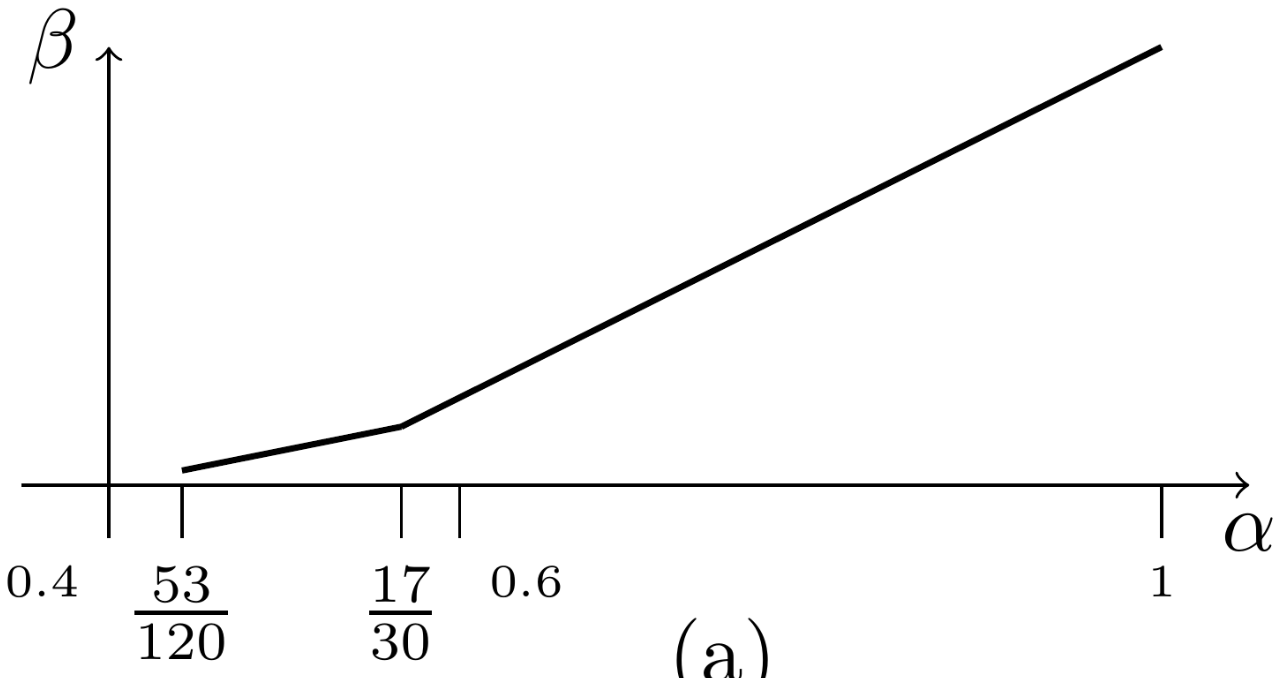
△-free graphs

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$$\frac{n}{2} = \lfloor \alpha n \rfloor \qquad \frac{n^2}{50} = \lfloor \beta n^2 \rfloor$$



$$\beta >_? \begin{cases} (2\alpha - 1)/4 & \text{when } 17/30 \le \alpha \le 1, \\ (5\alpha - 2)/25 & \text{when } 53/120 \le \alpha \le 17/30 \end{cases}$$



(a)

Sparse half

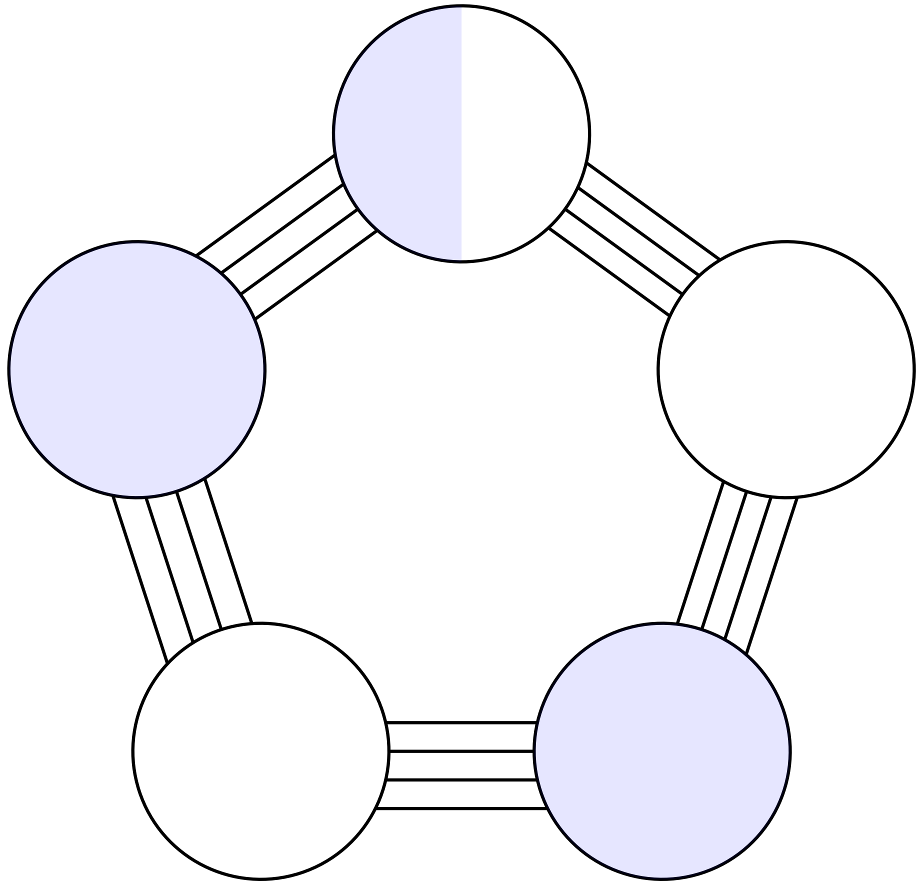
Erdős, 1976

△-free graphs

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\| \quad \|$

$\lfloor \alpha n \rfloor \quad \lfloor \beta n^2 \rfloor$



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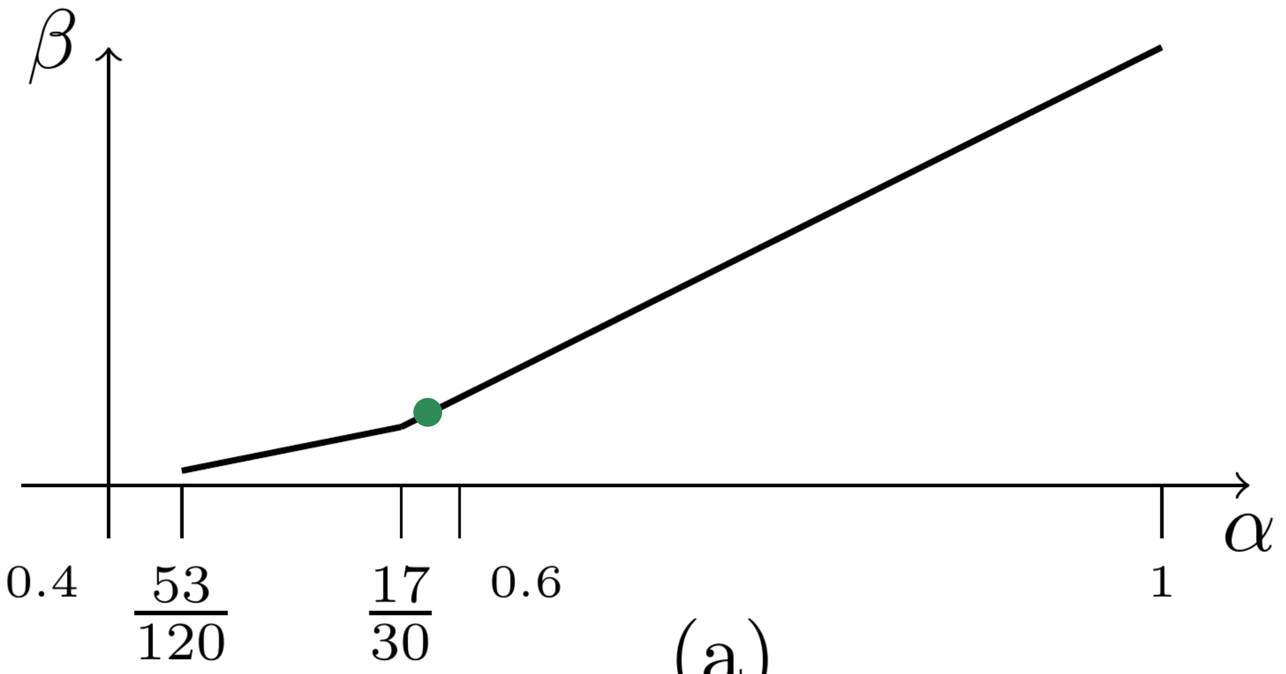


(a)

$\alpha \geq 0.579$

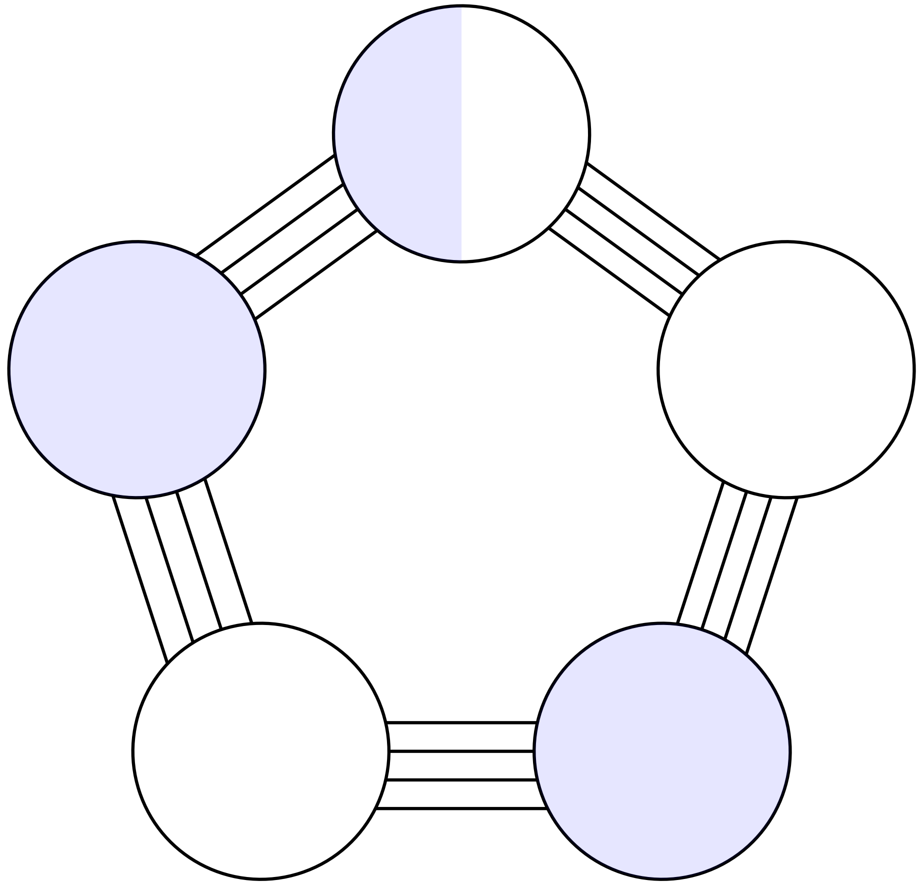
Sparse half

Erdős, 1976

△-free graphs

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\frac{n}{2} = \lfloor \alpha n \rfloor$, $\frac{n^2}{50} = \lfloor \beta n^2 \rfloor$



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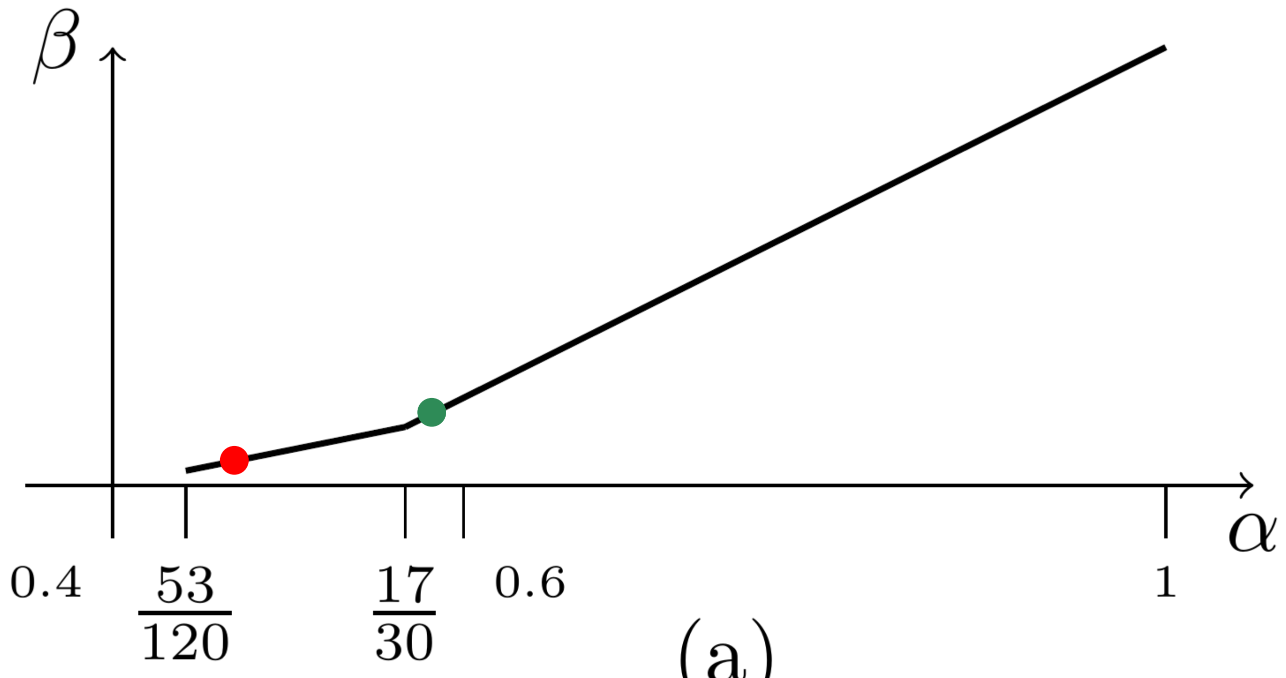


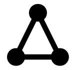
(a)

$\alpha \geq 0.579$

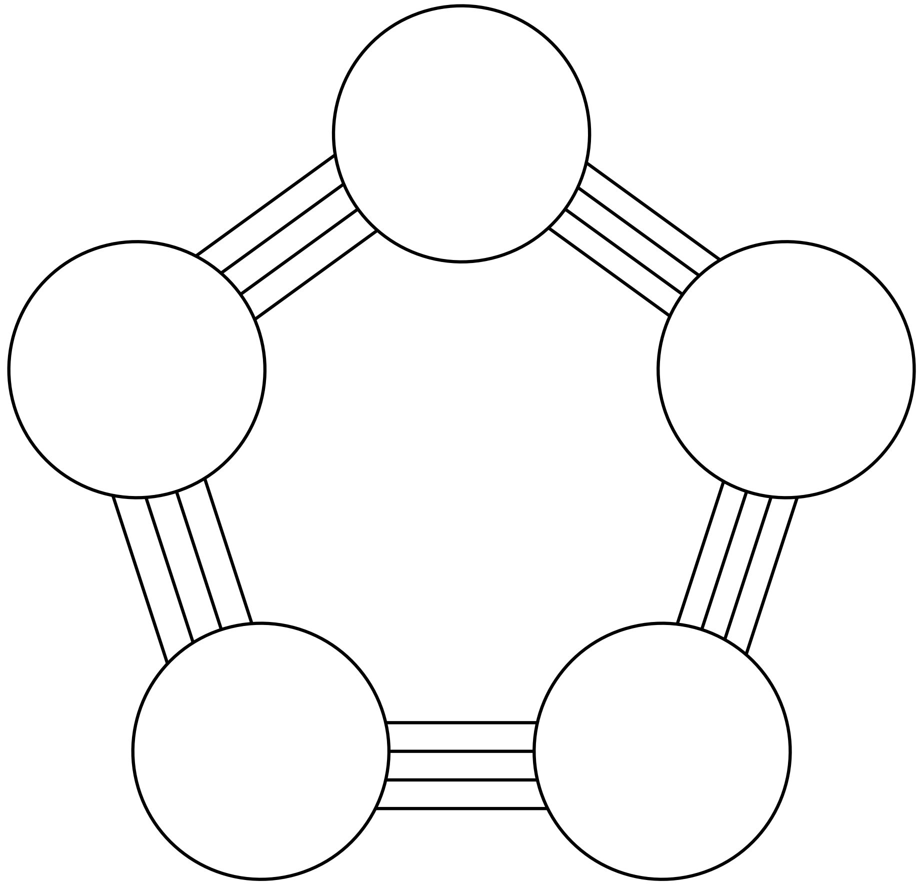
$\alpha < 0.474$

Bipartiteness

Erdős, 1976

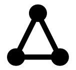
-free graphs

can be made bipartite by removing at most $\frac{n^2}{25}$ edges?



Bipartiteness

Erdős, 1976

-free graphs

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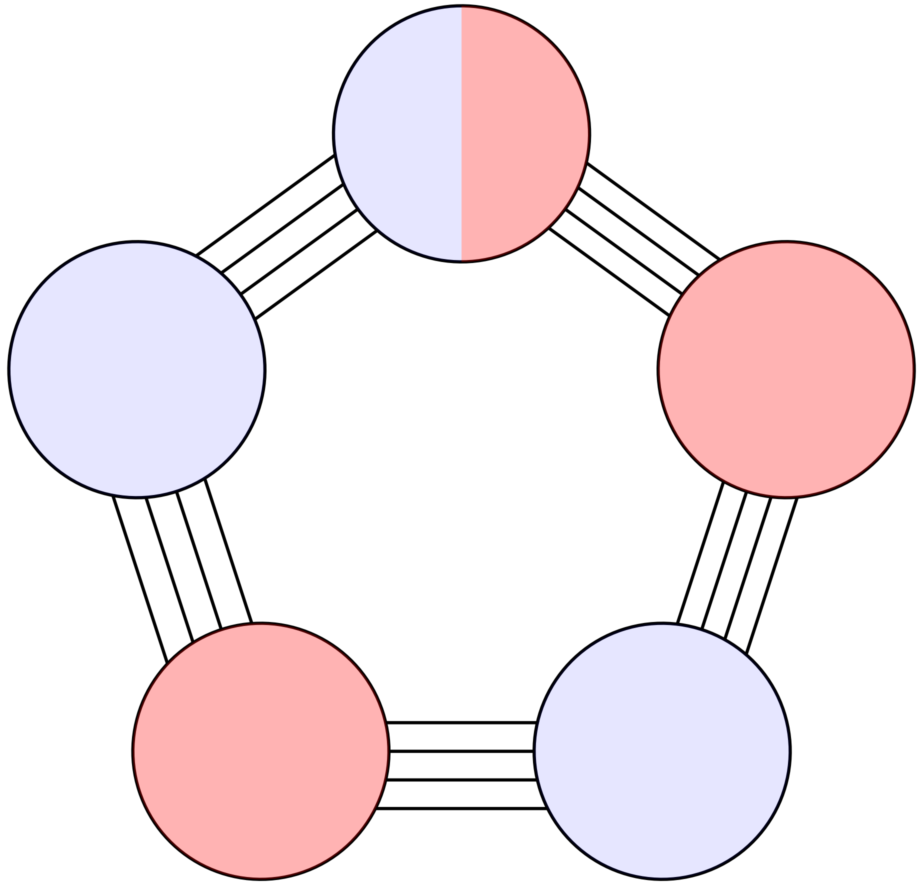
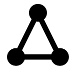


Bipartiteness

Erdős, 1976

 -free graphs

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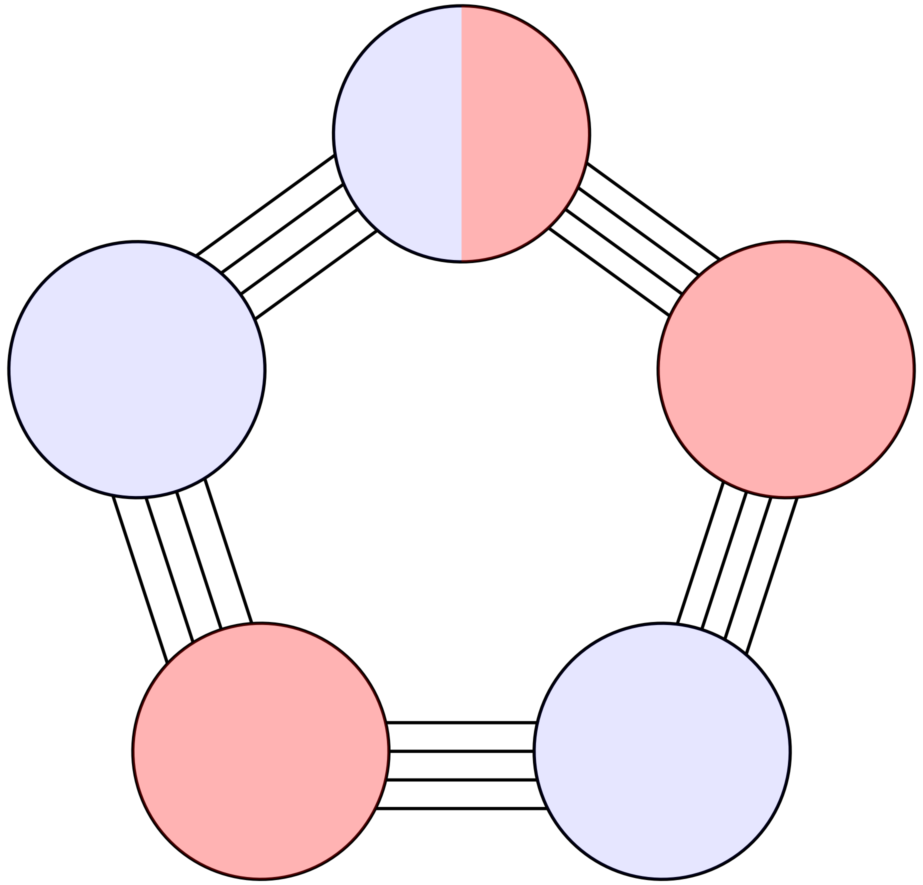


K_{r+1}-free graphs

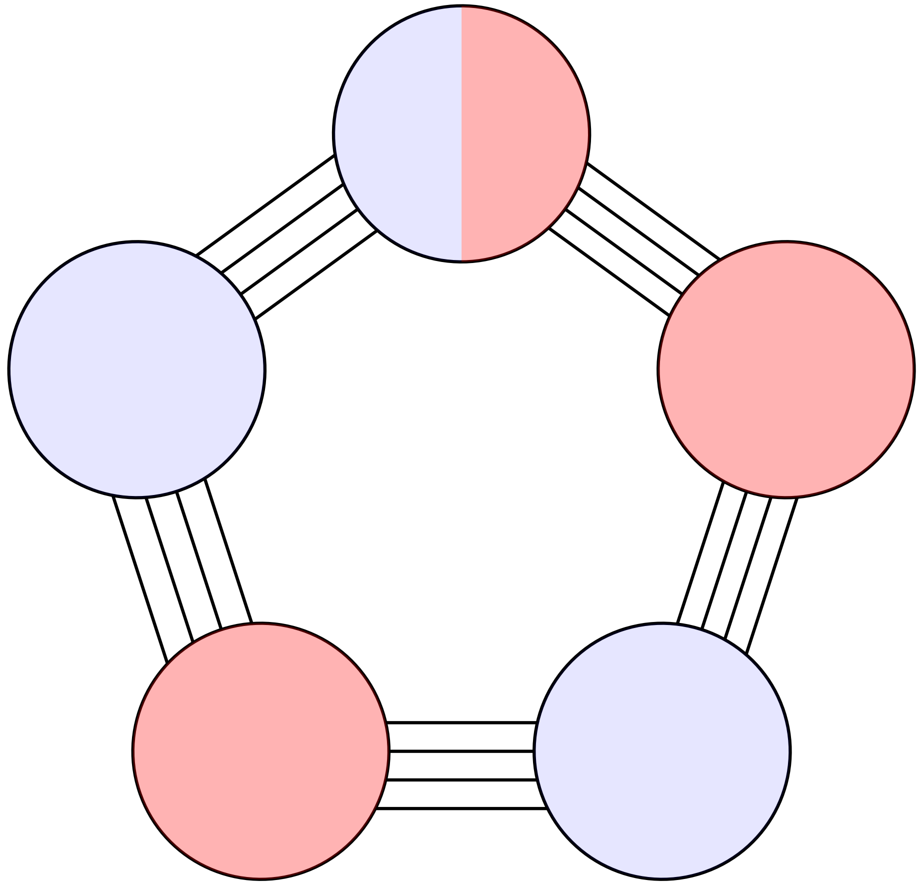
$$D_2(G) \leq_? \begin{cases} \frac{(r-1)^2}{4r^2} \cdot n^2 & r \text{ odd}, \\ \frac{r-2}{4r} \cdot n^2 & r \text{ even and } > 2, \\ \frac{n^2}{25} & r = 2. \end{cases}$$

Bipartiteness

Erdős, 1976

△-free graphs

can be made bipartite by removing at most $\frac{n^2}{25}$ edges?



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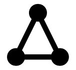
K_4-free

Bipartiteness

Erdős, 1976

-free graphs

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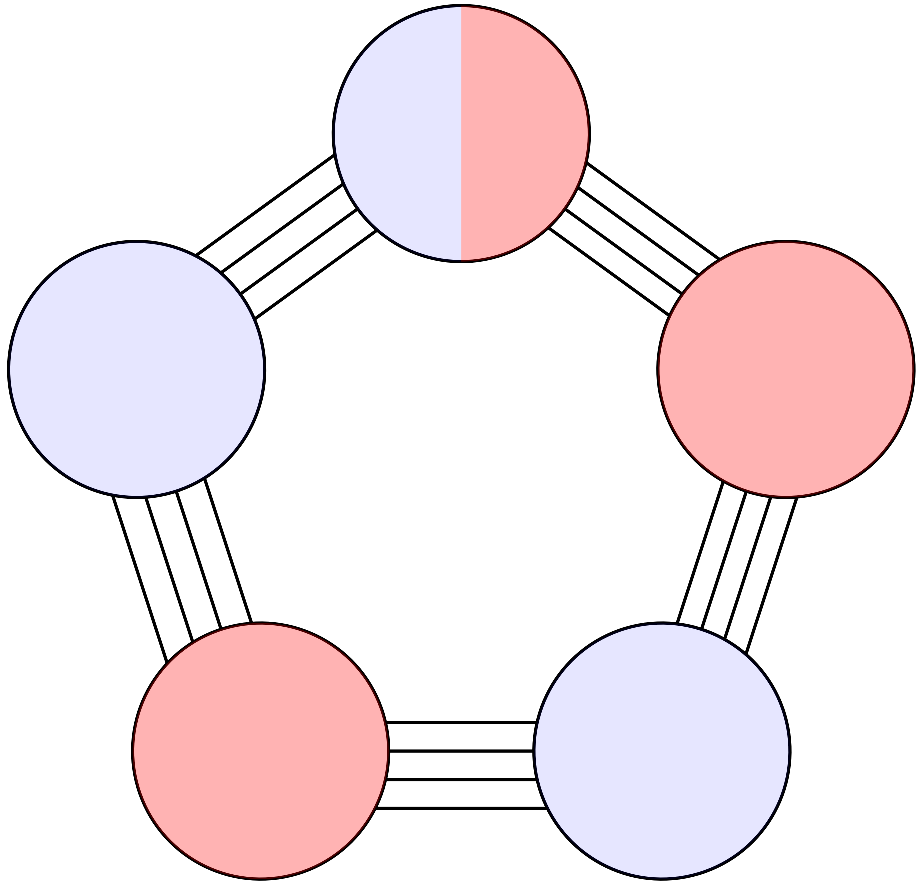

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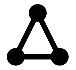
K_4-free

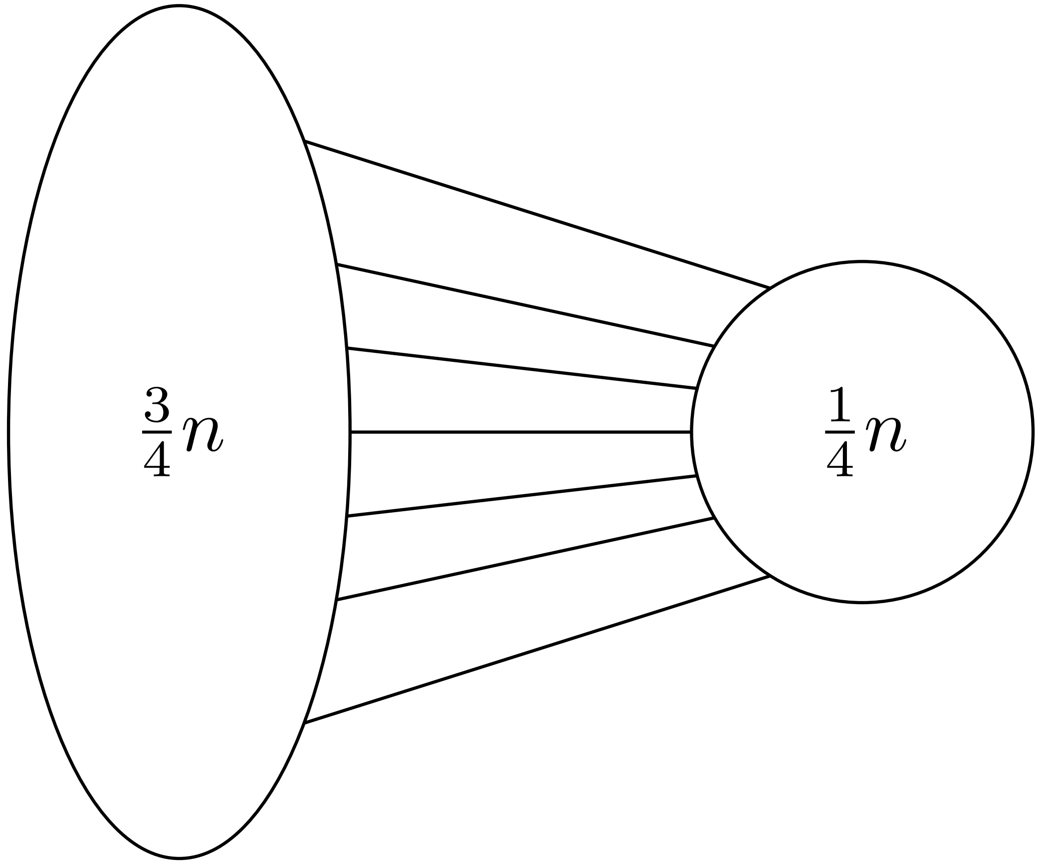
r even is harder than odd

Balanced bipartiteness

Balogh, Clemen, Lidický, 2022

-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

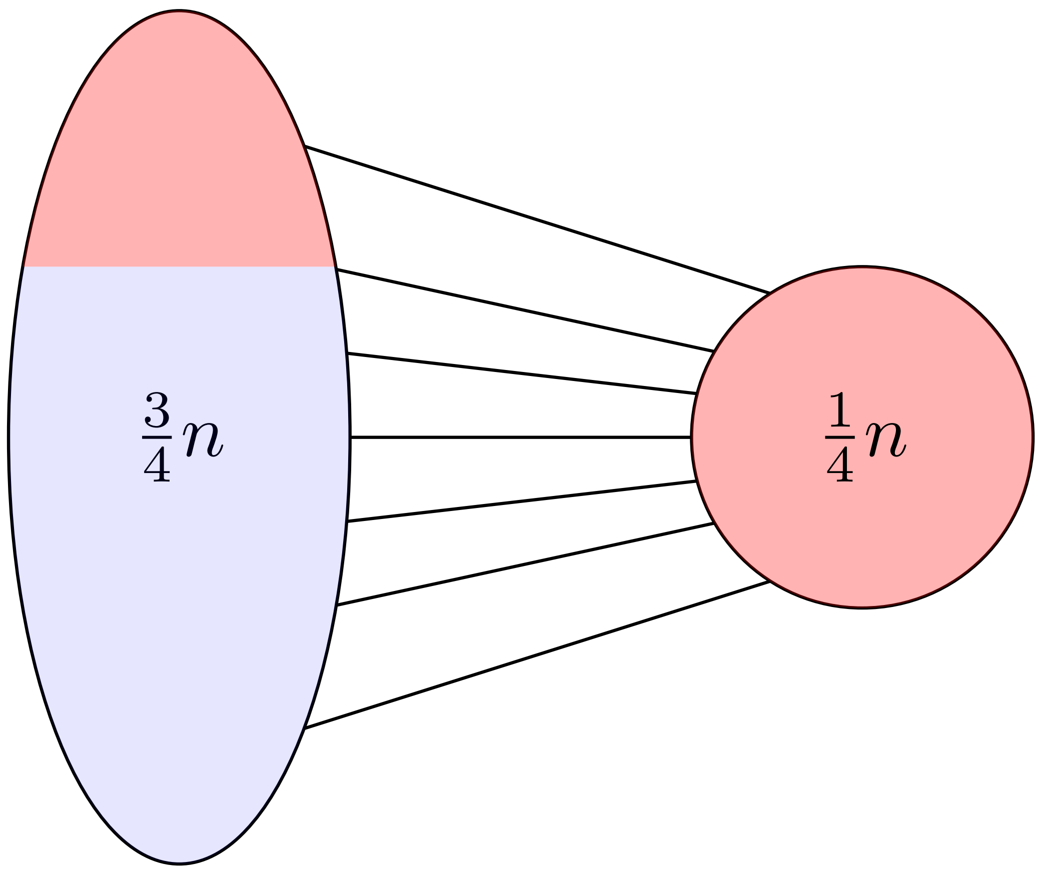


Balanced bipartiteness

Balogh, Clemen, Lidický, 2022

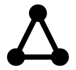
$\triangle$-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

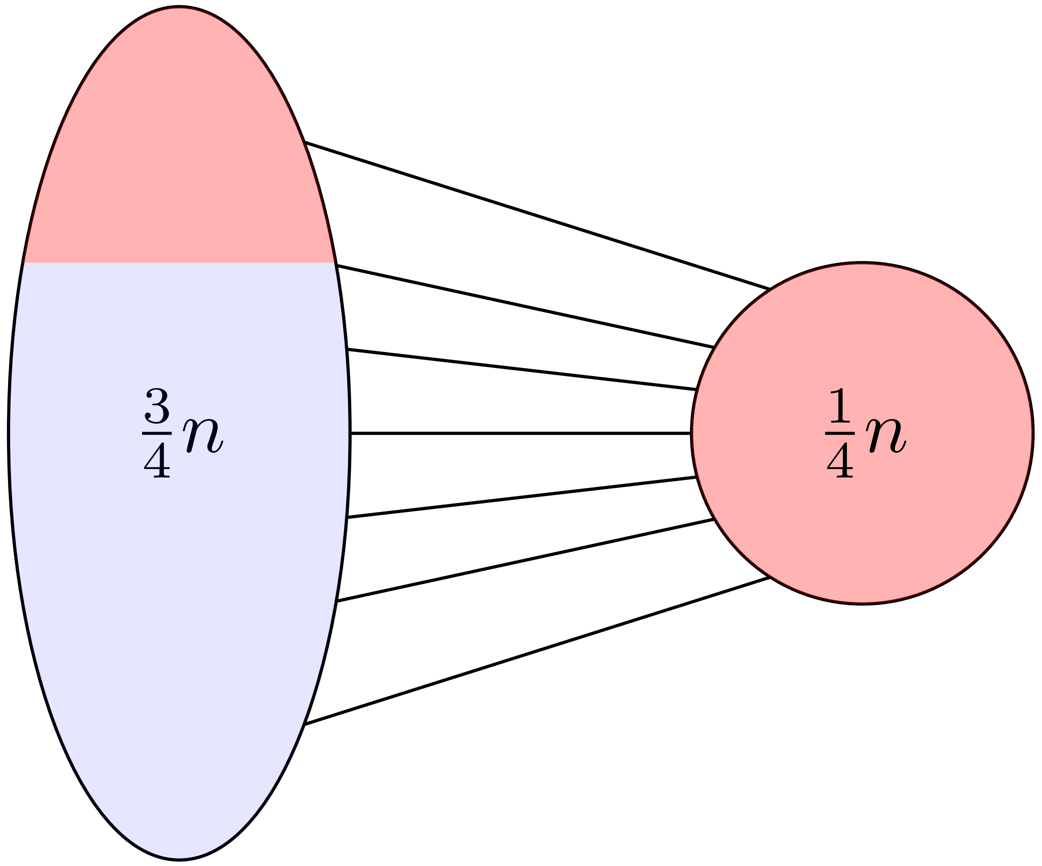


Balanced bipartiteness

Balogh, Clemen, Lidický, 2022

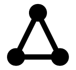
-free graphs

have subset A of size $\frac{n}{2}$ s. t. $e(A) + e(A^c) \leq \frac{n^2}{16}$



Balanced bipartiteness

Balogh, Clemen, Lidický, 2022

-free graphs

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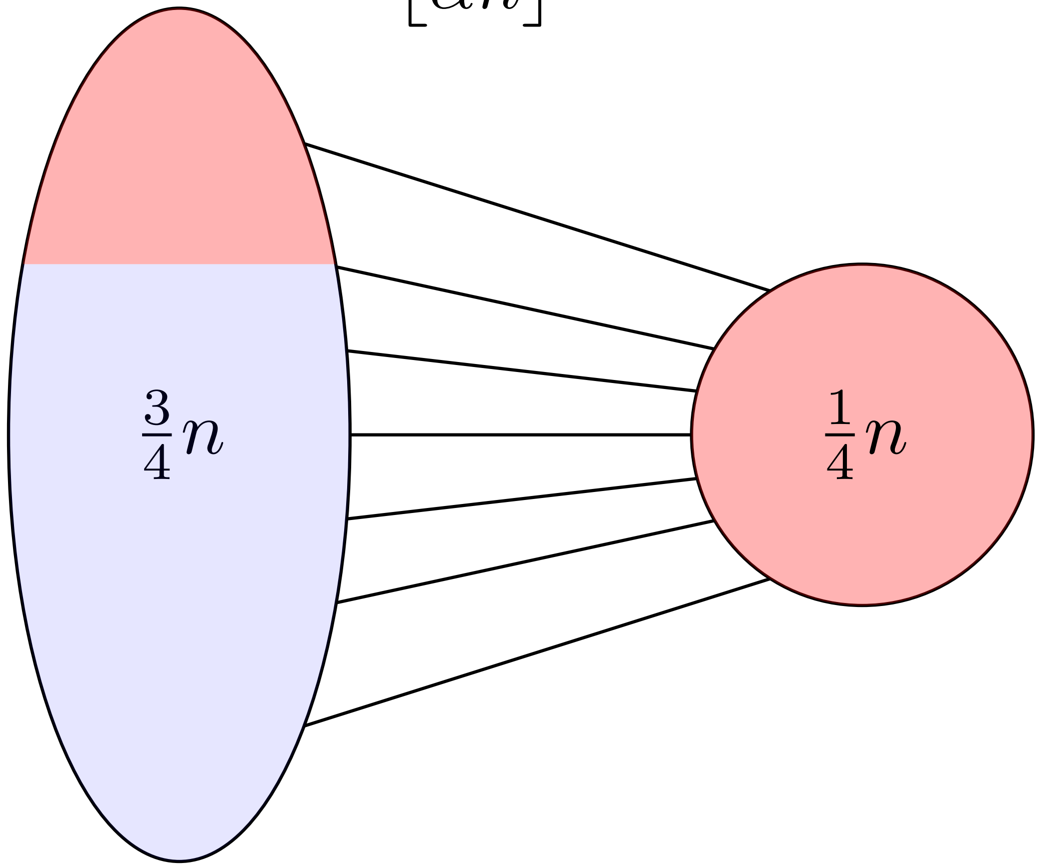

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Balogh, Clemen, Lidický, 2022

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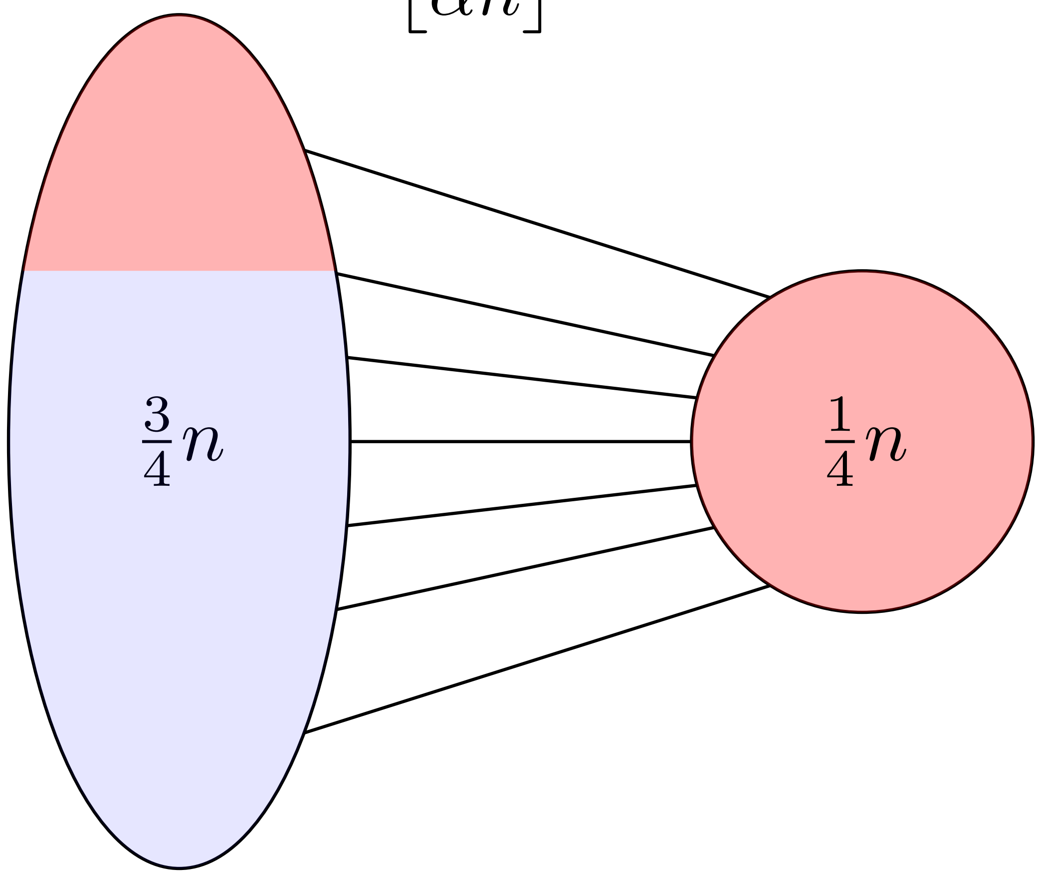
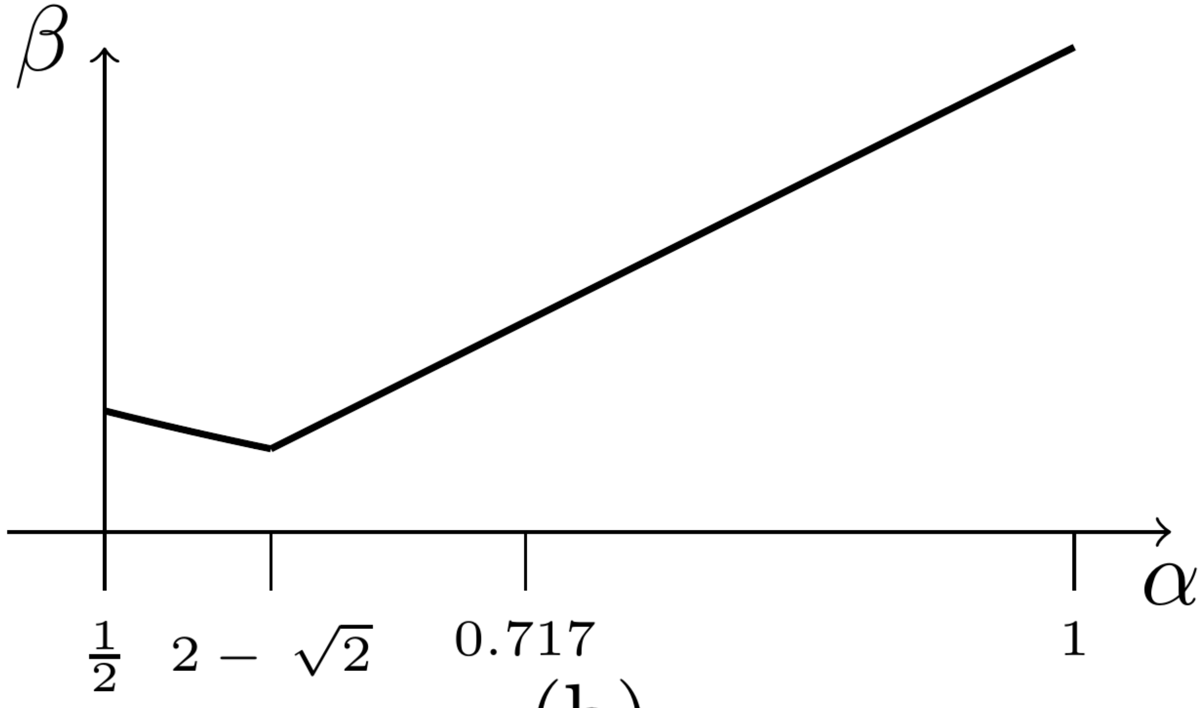


$$\beta > \begin{cases} (2\alpha - 1)/4 & \text{when } 2 - \sqrt{2} \leq \alpha \leq 1, \\ (1-\alpha)^2/4 & \text{when } \frac{1}{2} \leq \alpha < 2 - \sqrt{2} \end{cases}$$



(b)

Balanced bipartiteness

Balogh, Clemen, Lidický, 2022

$\triangle$-free graphs

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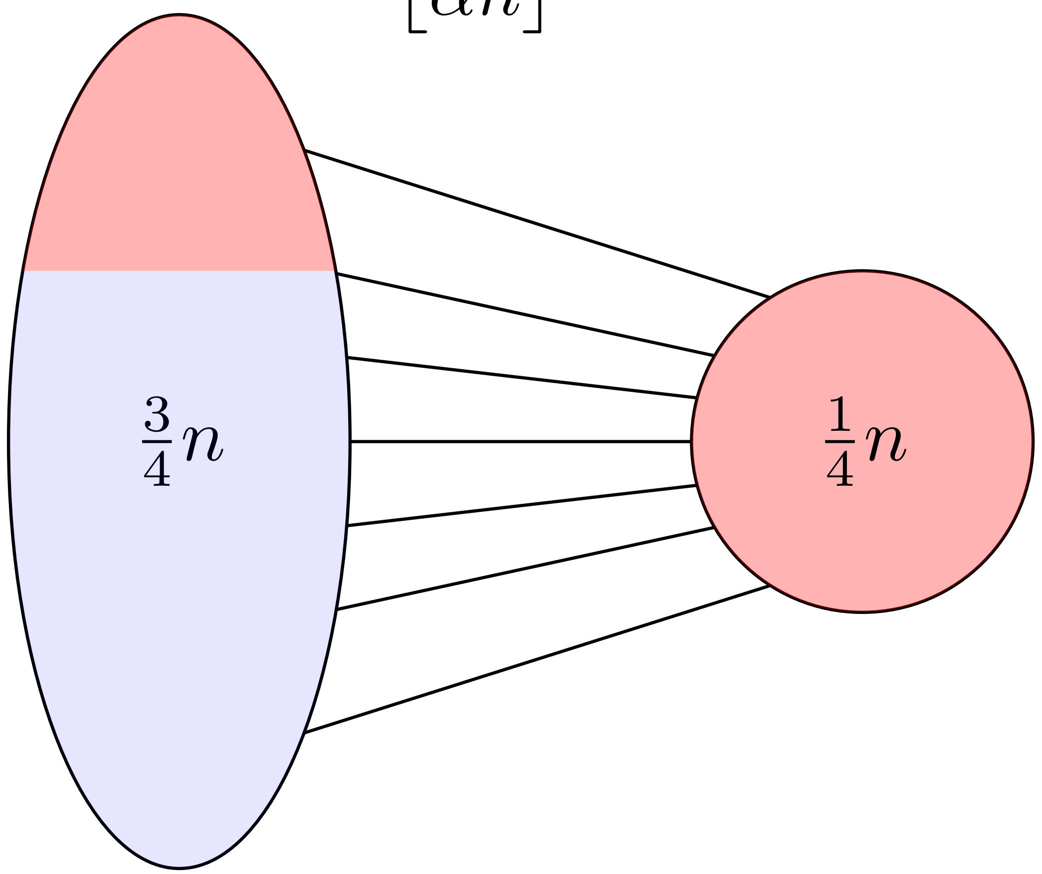

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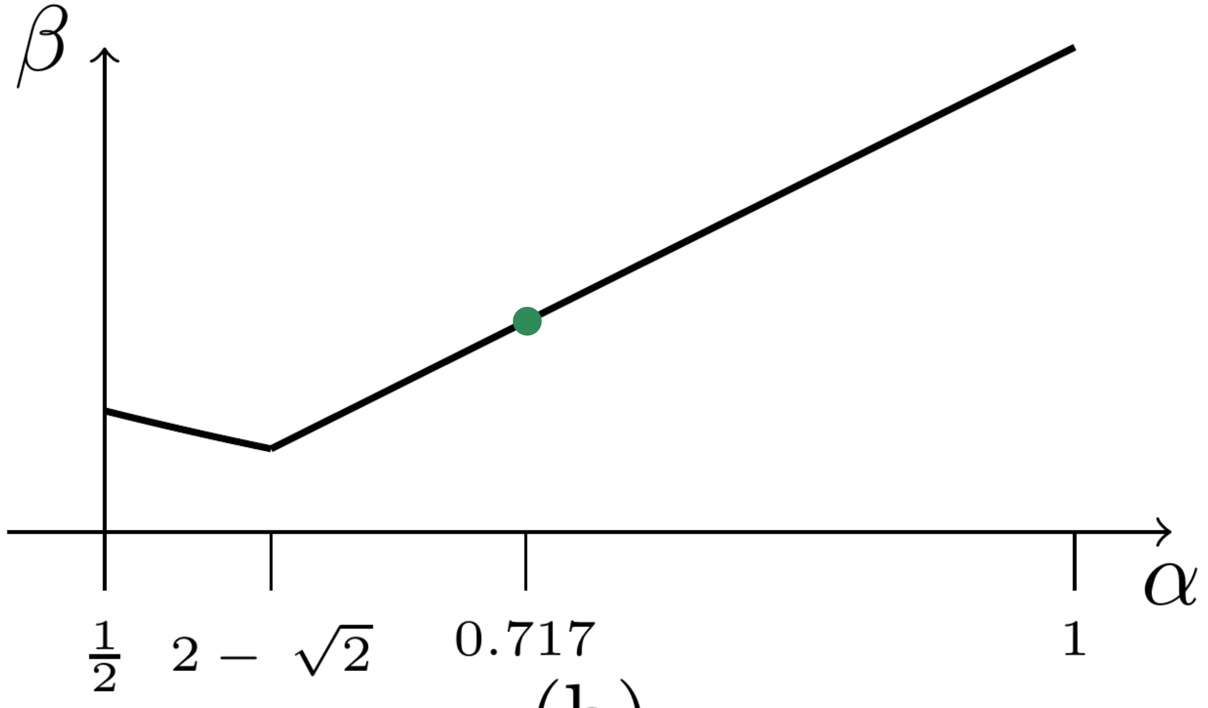
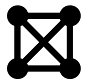
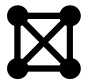


(b)

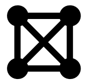
$\alpha \ge 0.717$

 -free

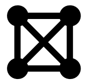
	Sparse half	Make bipartite	Make balanced bipartite
 -free			
 -free			

 -free

	Sparse half	Make bipartite	Make balanced bipartite
 -free	$\frac{n^2}{25} \cdot \frac{1}{2}$	$\frac{n^2}{25}$	$\frac{n^2}{16}$
 -free			

 -free

	Sparse half	Make bipartite	Make balanced bipartite
 -free	$\frac{n^2}{25} \cdot \frac{1}{2}$	$\frac{n^2}{25}$	$\frac{n^2}{16}$
 -free	$\frac{n^2}{9} \cdot \frac{1}{2}$	$\frac{n^2}{9}$	$\frac{n^2}{9}$

 -free

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 -free	$\frac{n^2}{25} \cdot \frac{1}{2}$	$\frac{n^2}{25}$	$\frac{n^2}{16}$
 -free	$\frac{n^2}{9} \cdot \frac{1}{2}$	$\frac{n^2}{9}$	$\frac{n^2}{9}$

Flag algebras

Graph limits

$$f(x) = -\frac{x^4}{4} + 7x$$

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$f'(x) = -x^3 + 7$

$x = \sqrt[3]{7}$

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$f_{max} = \frac{21\sqrt[3]{7}}{4}$

Graph limits

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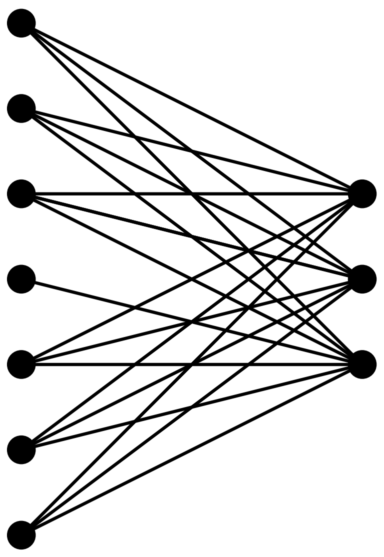

0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	0	0	0

Graph limits

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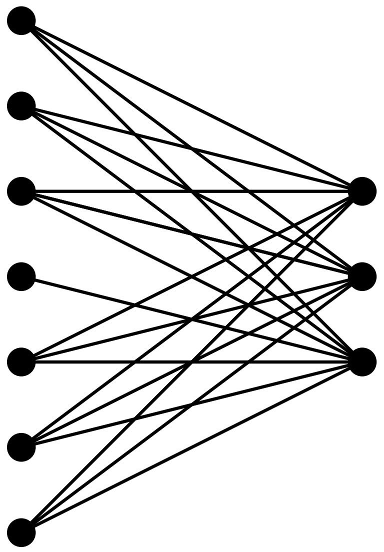

0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
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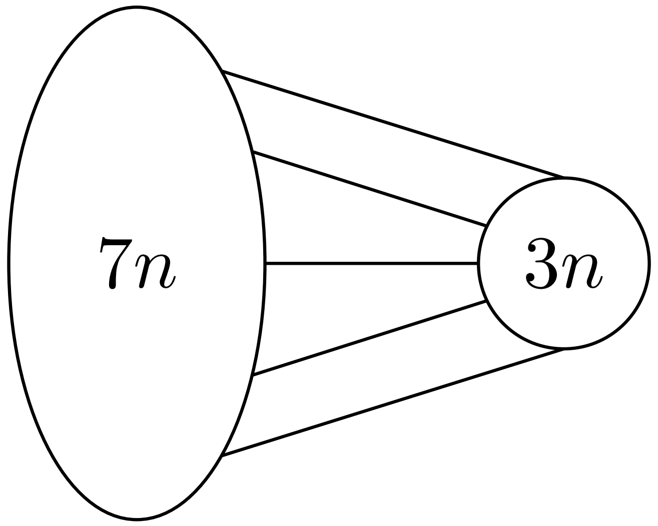
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	0	0	0



0	1
1	0

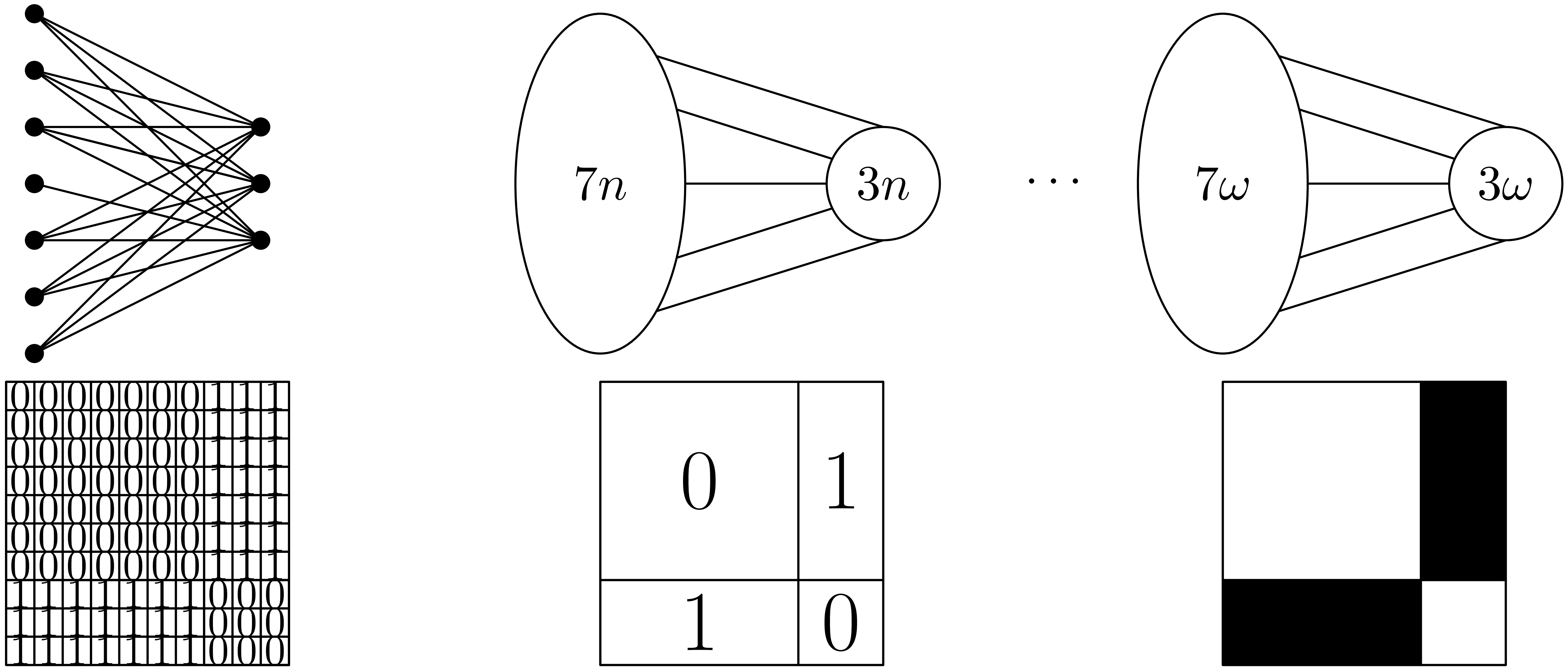
Graph limits

$$f(x) = -\frac{x^4}{4} + 7x$$

$$f'(x) = -x^3 + 7$$

$$x = \sqrt[3]{7}$$

$$f_{max} = \frac{21\sqrt[3]{7}}{4}$$



Graph limits

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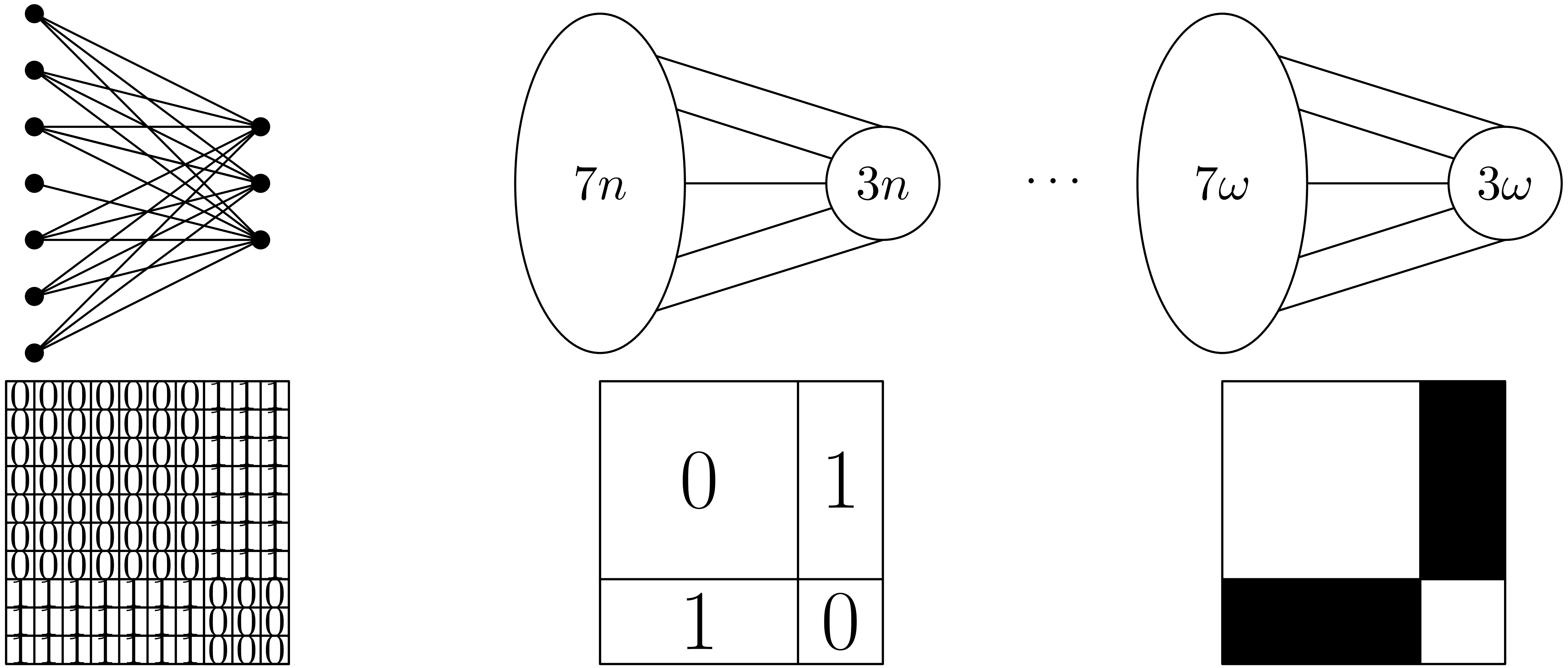


$$W : [0,1]^2 \mapsto [0,1]$$

$$W(x,y) = W(y,x)$$

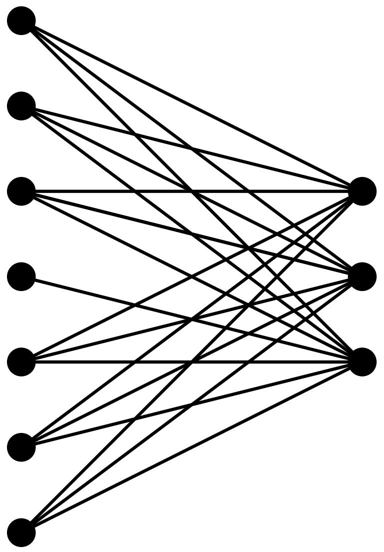
Graph limits

$f(x) = -\frac{x^4}{4} + 7x$

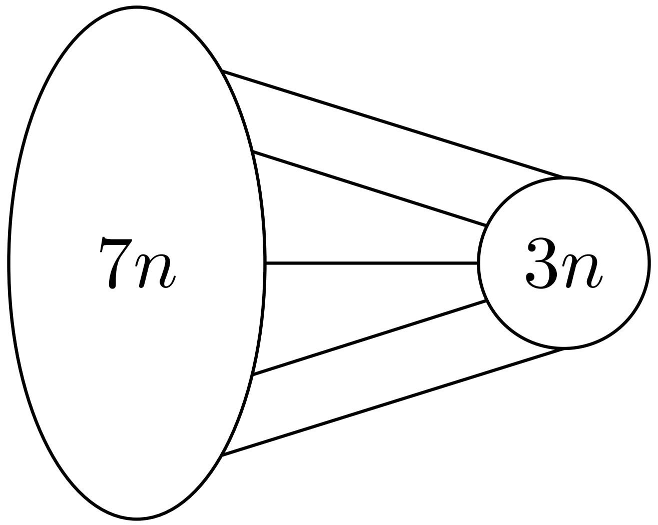
$f'(x) = -x^3 + 7$

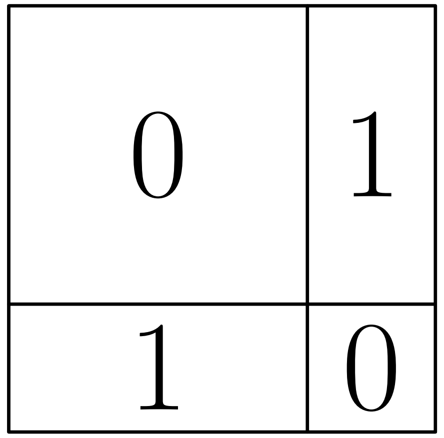
$x = \sqrt[3]{7}$

$f_{max} = \frac{21\sqrt[3]{7}}{4}$

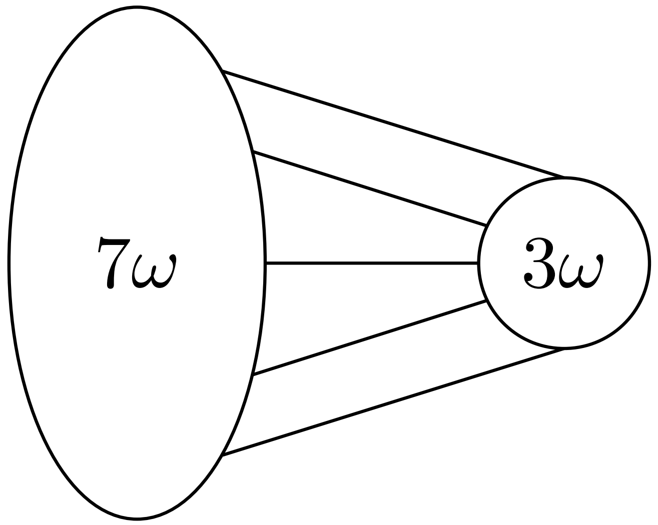


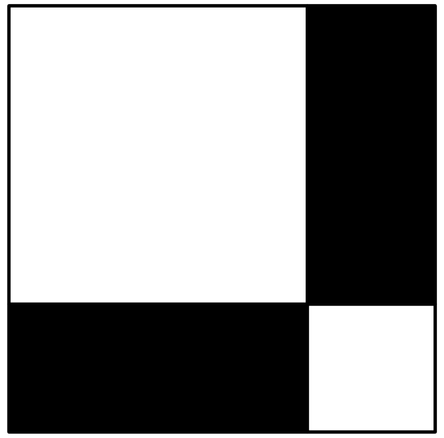
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
0	0	0	0	0	0	0	1	1	1
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	0	0	0
1	1	1	1	1	1	1	0	0	0





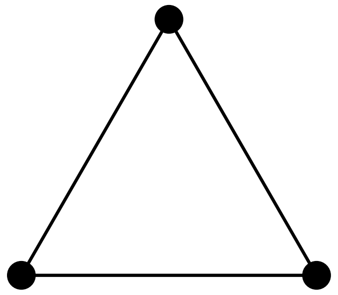
$\cdots$





$W : [0,1]^2 \mapsto [0,1]$

$W(x,y) = W(y,x)$



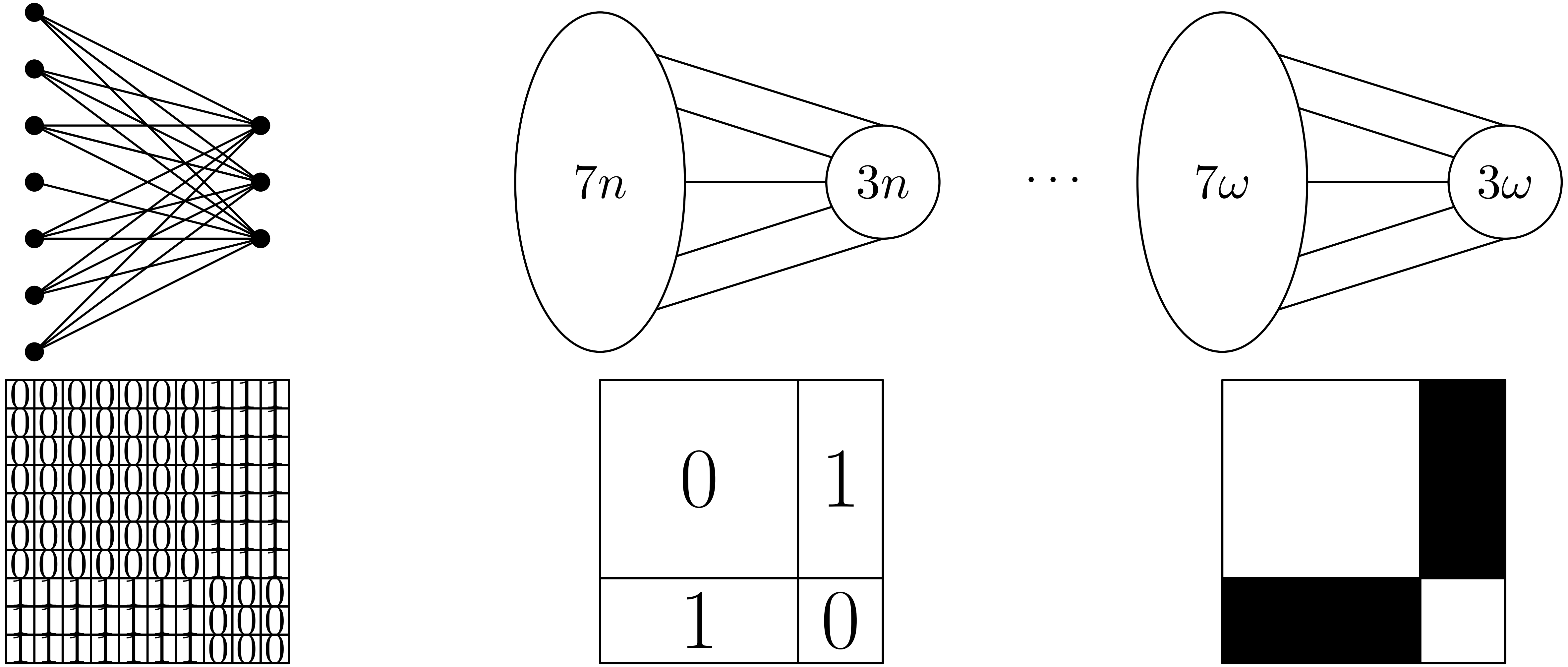
Graph limits

$f(x) = -\frac{x^4}{4} + 7x$

$f'(x) = -x^3 + 7$

$x = \sqrt[3]{7}$

$f_{max} = \frac{21\sqrt[3]{7}}{4}$



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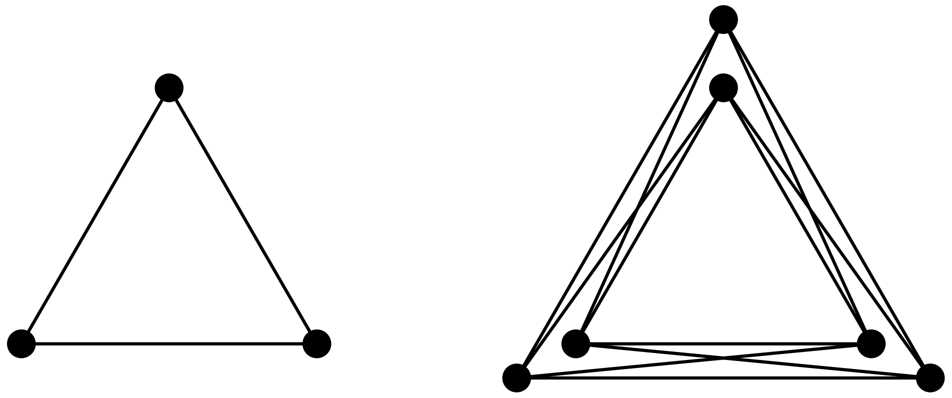

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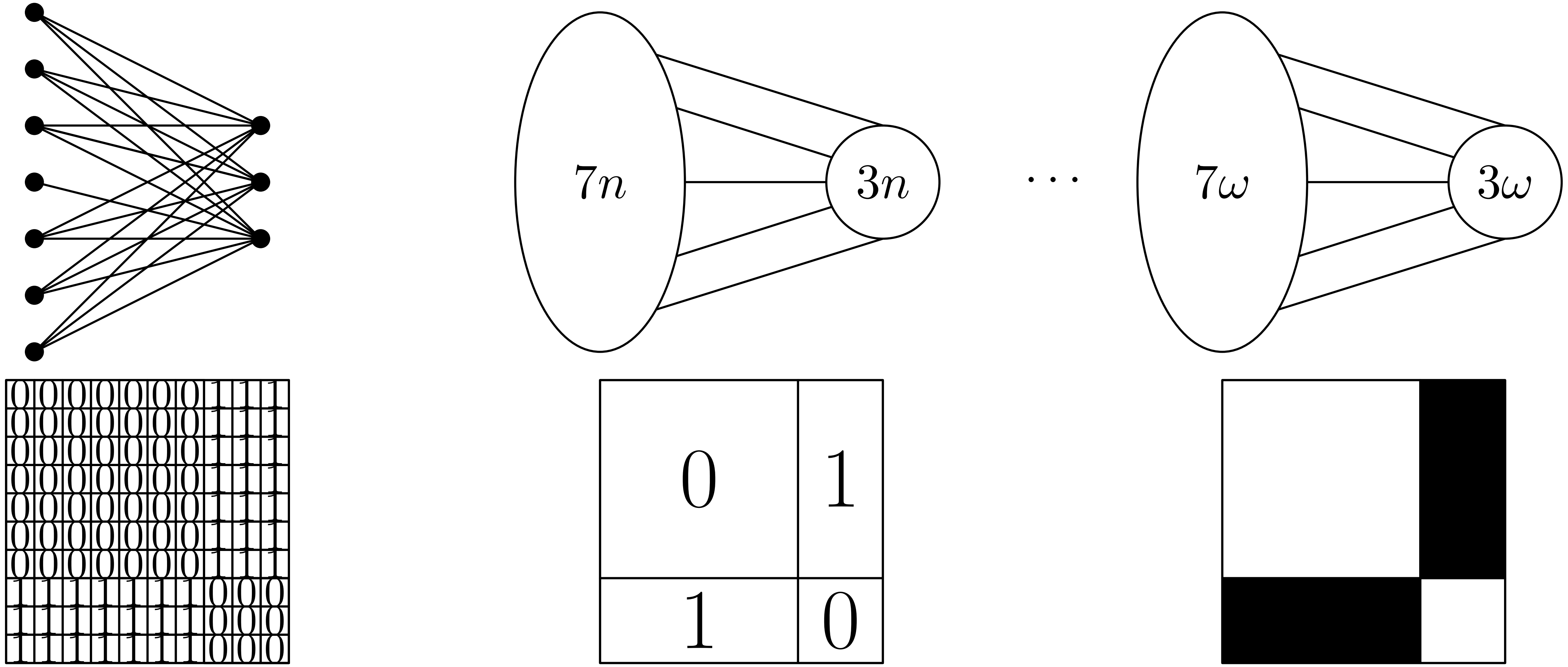

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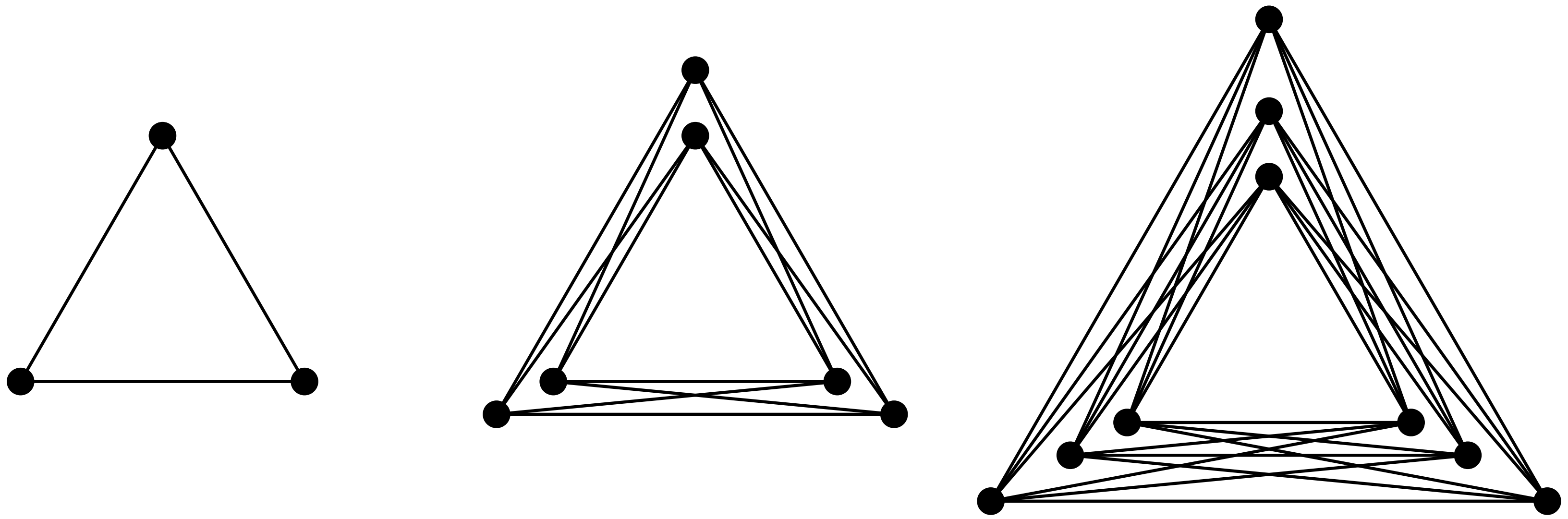

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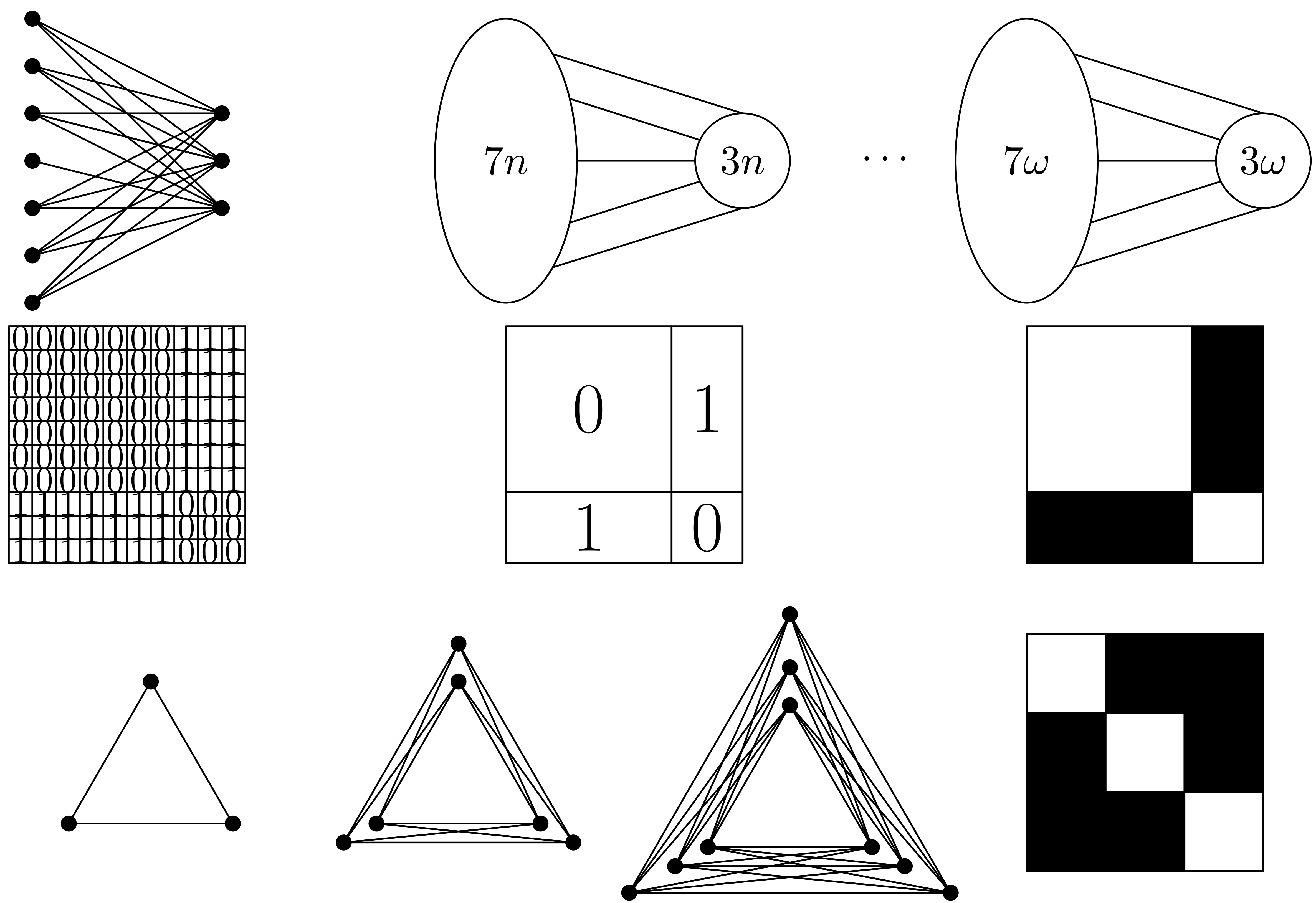

$$W : [0,1]^2 \mapsto [0,1]$$

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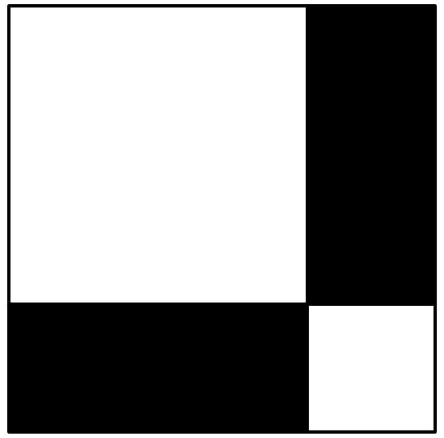
Flags

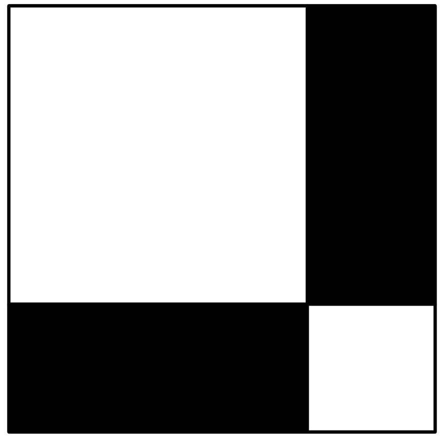
$d(G, W)$ - density of graph G in graphon W



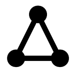
$d($$, W) = \frac{42}{100}$

Flags

$d(G, W)$ - density of graph G in graphon W

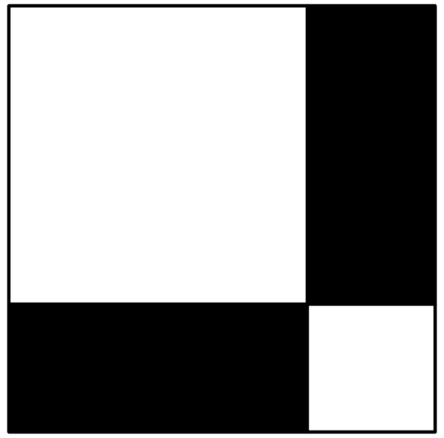


$d($  $, W) = \frac{42}{100}$

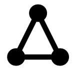
$d($  $, W) = 0$

Flags

$d(G, W)$ - density of graph G in graphon W

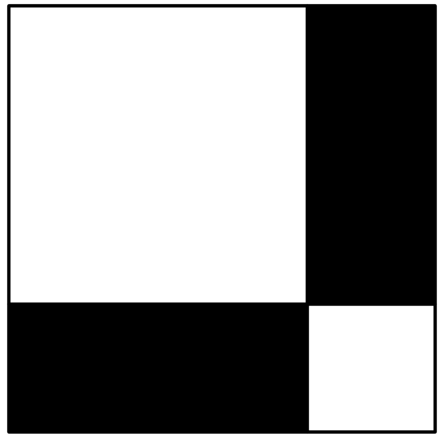


$d($$, W) = \frac{42}{100}$  $= \frac{42}{100}$

$d($$, W) = 0$  $= 0$

Flags

$d(G,W)$ - density of graph G in graphon W



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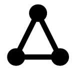
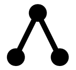
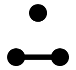
$d($$, W) = 0$  $= 0$

 $=$  $+ \frac{2}{3}$  $+ \frac{1}{3}$ 

Flags

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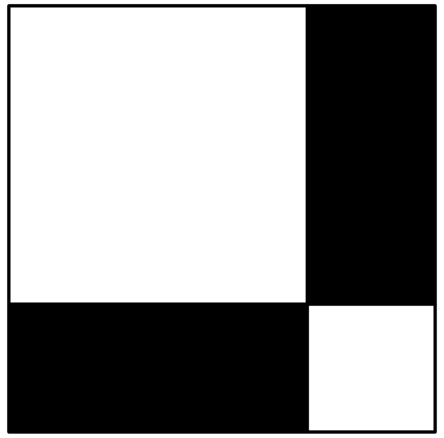
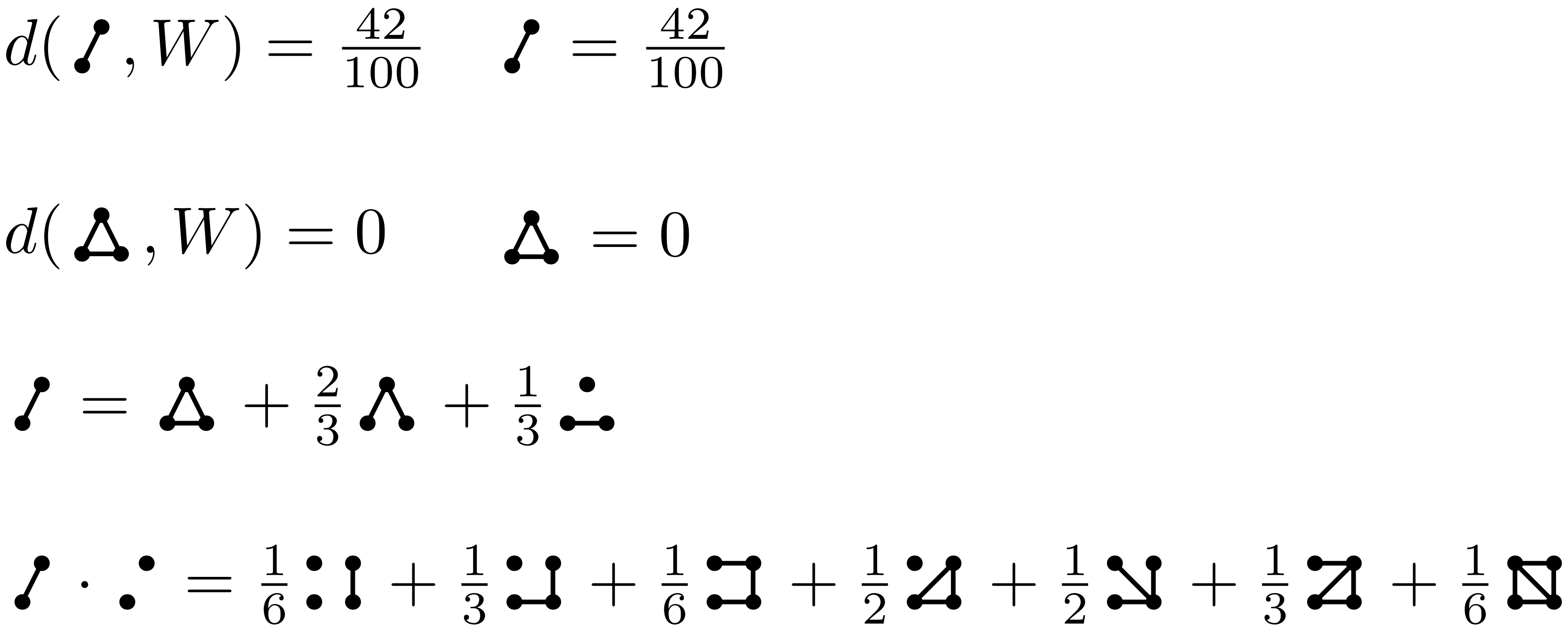
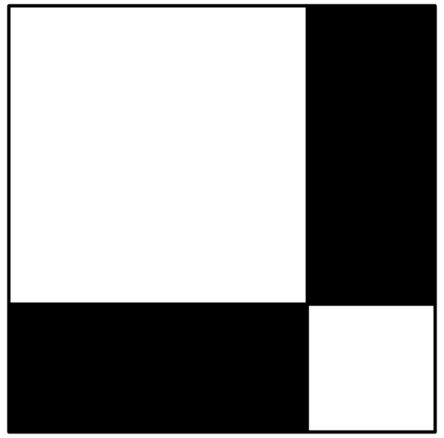
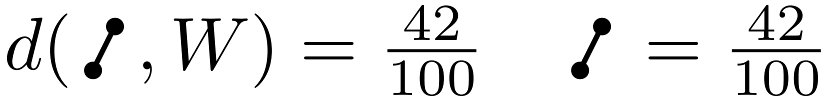


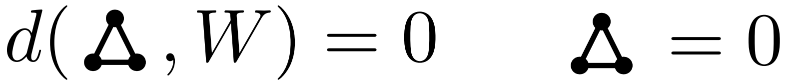


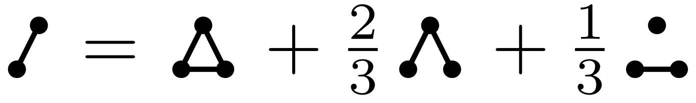
Flags

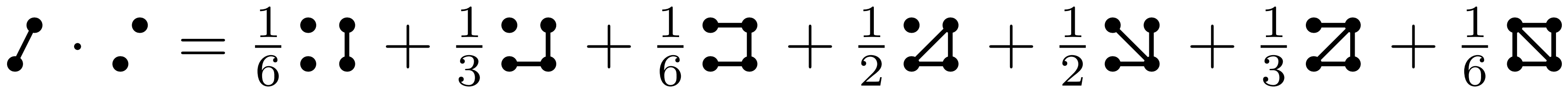
$d(G, W)$ - density of graph G in graphon W

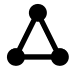






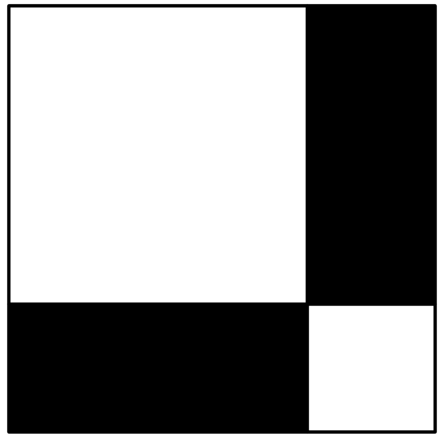


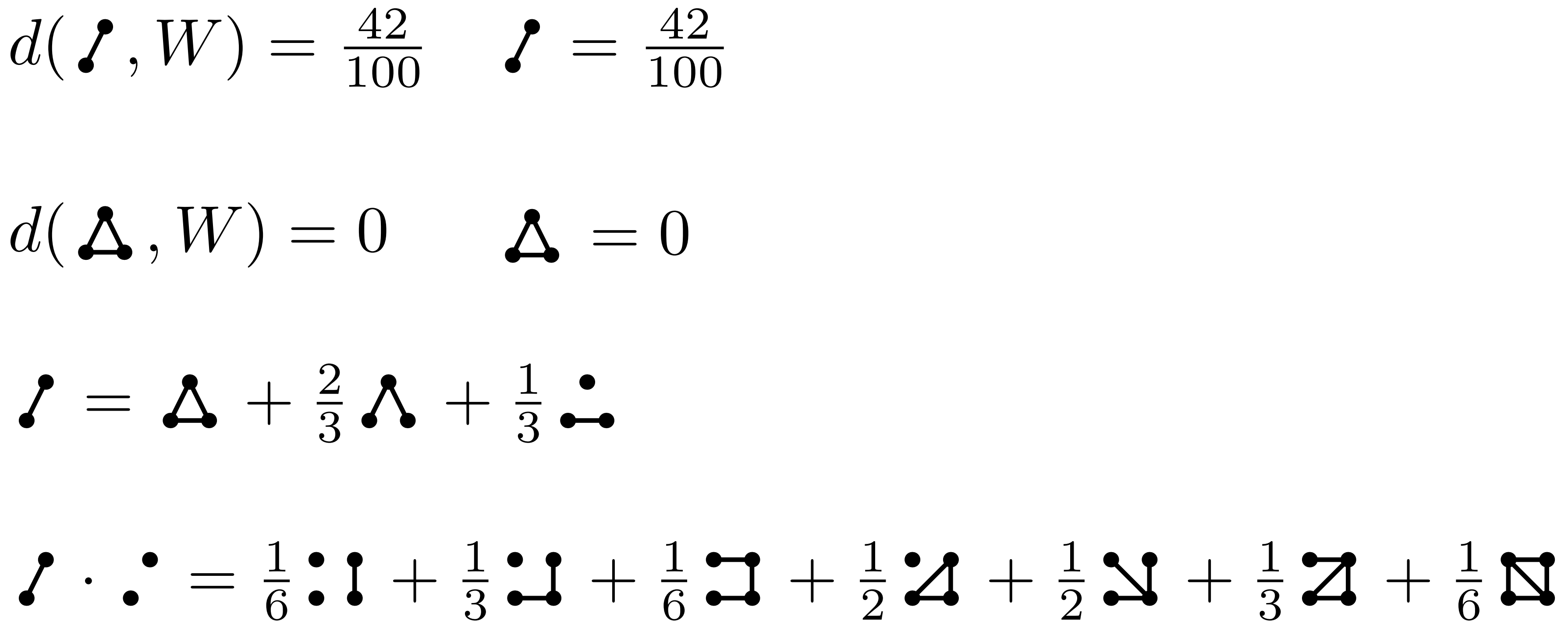


-free graphs have at most $\frac{n^2}{4}$ edges.

Flags

$d(G, W)$ - density of graph G in graphon W

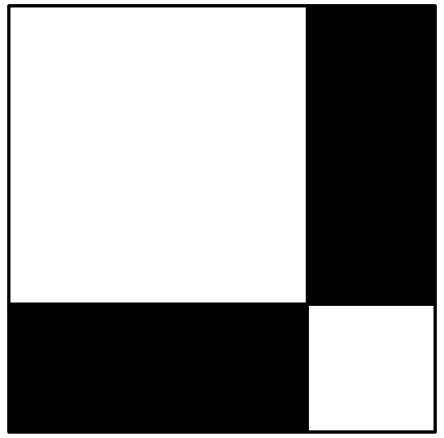


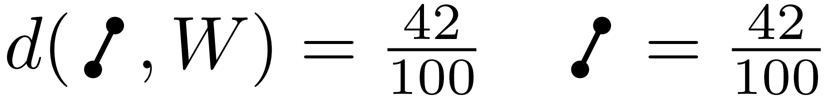


-free graphs have at most $\frac{n^2}{4}$ edges. $= 0 \rightarrow \ \leq \frac{1}{2}$

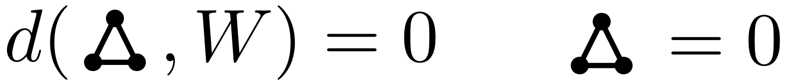
Flags

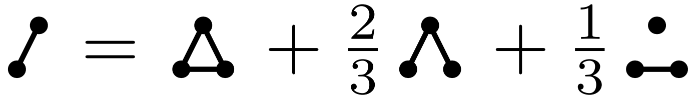
$d(G, W)$ - density of graph G in graphon W

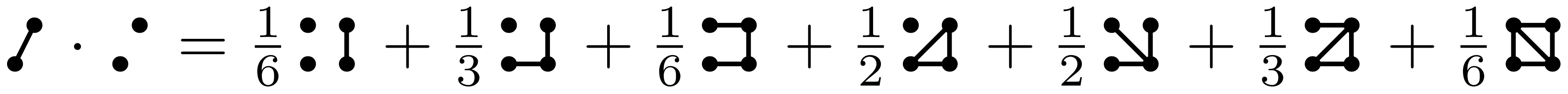


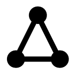
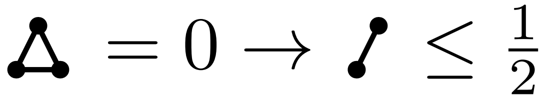


$\Delta \leq \frac{n}{2}$



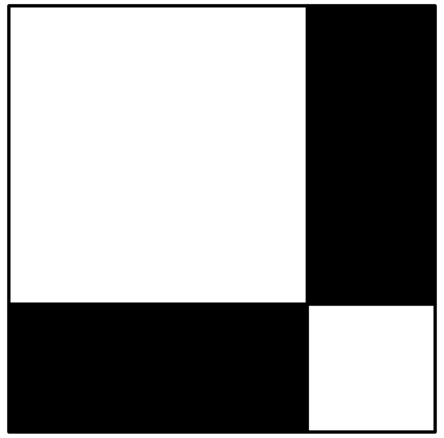




-free graphs have at most $\frac{n^2}{4}$ edges. 

Flags

$d(G, W)$ - density of graph G in graphon W



$d(\;, W) = \frac{42}{100}$ $\quad = \frac{42}{100}$ $\qquad \Delta \leq \frac{n}{2}$

$d(\;, W) = 0$ $\quad = 0$ $\qquad \leq \frac{1}{2}$

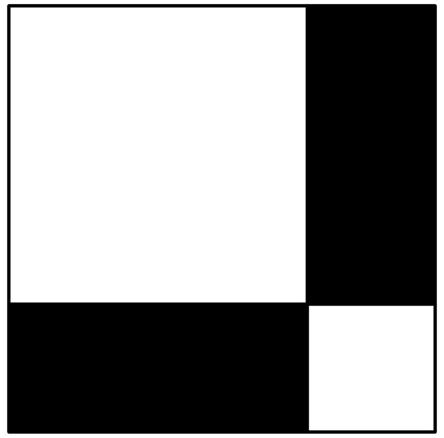
$= \; + \frac{2}{3} \; + \frac{1}{3}$

$\cdot \; = \frac{1}{6} \; + \frac{1}{3} \; + \frac{1}{6} \; + \frac{1}{2} \; + \frac{1}{2} \; + \frac{1}{3} \; + \frac{1}{6}$

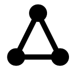
-free graphs have at most $\frac{n^2}{4}$ edges. $= 0 \to \; \leq \frac{1}{2}$

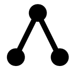
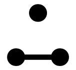
Flags

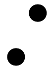
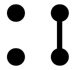
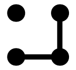
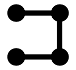
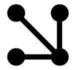
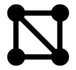
$d(G, W)$ - density of graph G in graphon W

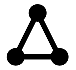
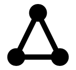


$d($$, W) = \frac{42}{100}$  $= \frac{42}{100}$

$d($$, W) = 0$  $= 0$

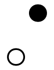
 $=$  $+ \frac{2}{3}$  $+ \frac{1}{3}$ 

 $\cdot$  $= \frac{1}{6}$  $+ \frac{1}{3}$  $+ \frac{1}{6}$  $+ \frac{1}{2}$  $+ \frac{1}{2}$  $+ \frac{1}{3}$  $+ \frac{1}{6}$ 

-free graphs have at most $\frac{n^2}{4}$ edges.  $= 0 \rightarrow$  $\leq \frac{1}{2}$

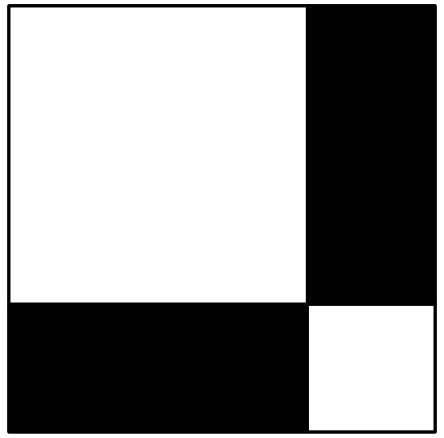
$\Delta \leq \frac{n}{2}$

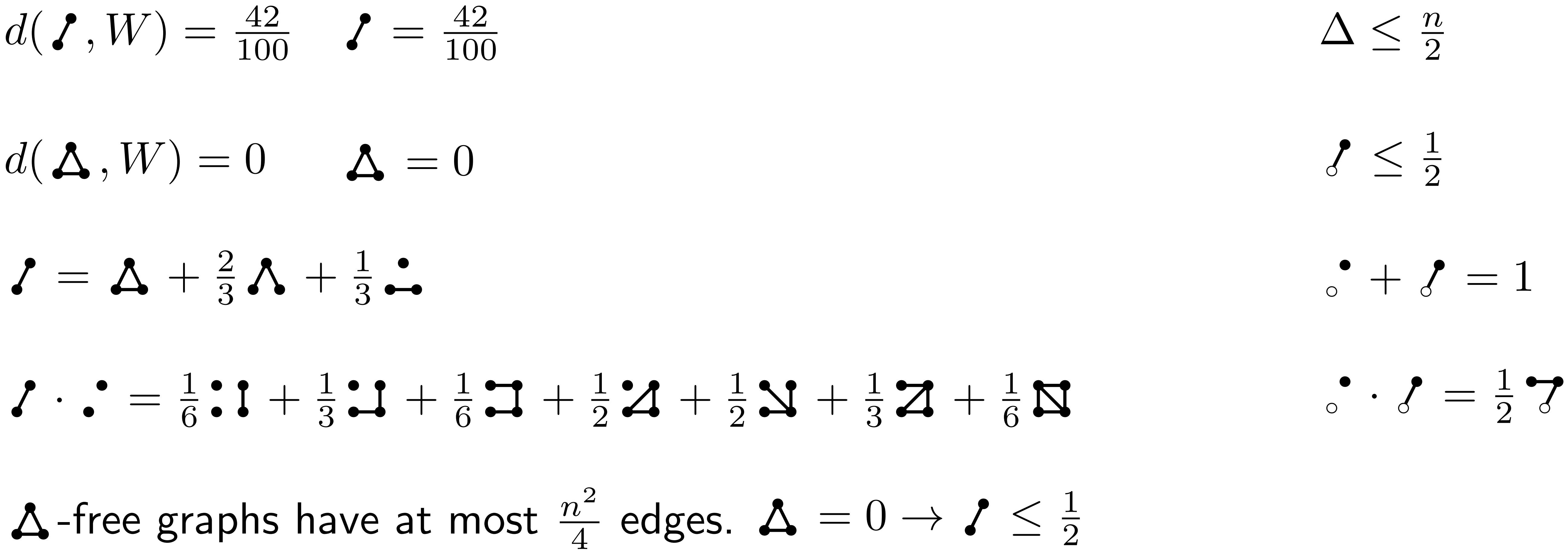
 $\leq \frac{1}{2}$

 $+$  $= 1$

Flags

$d(G, W)$ - density of graph G in graphon W





Flags

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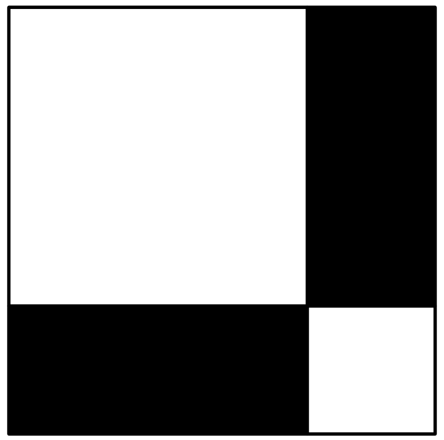
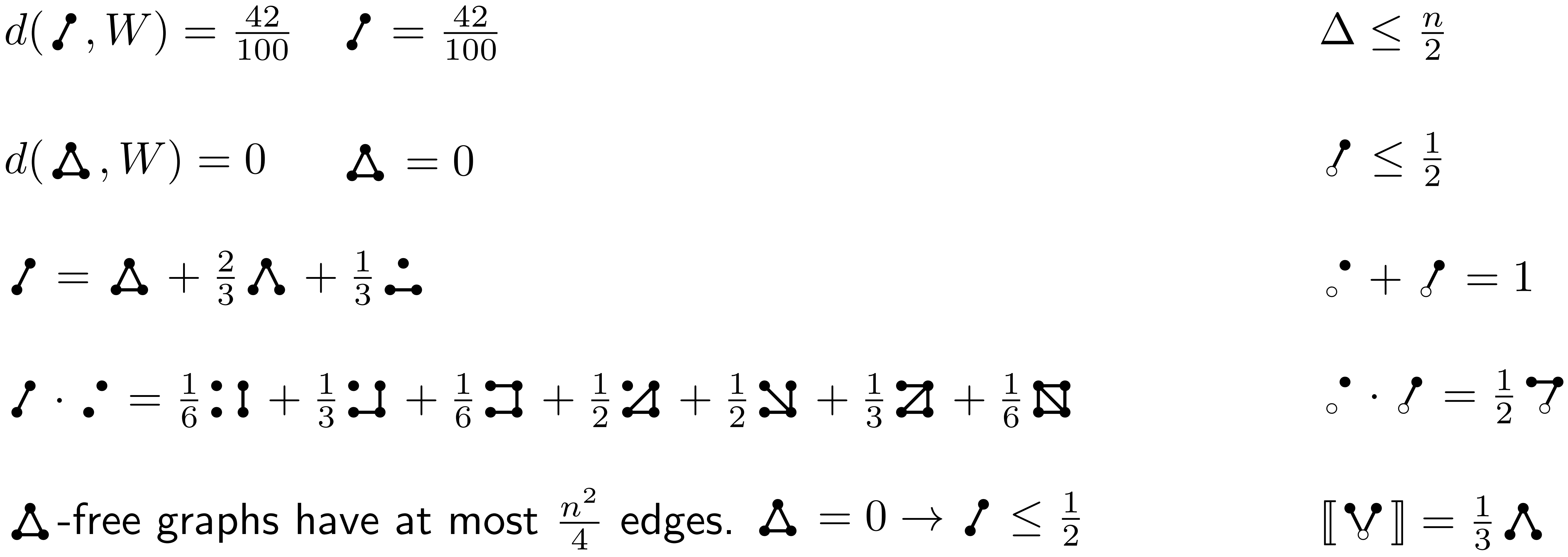
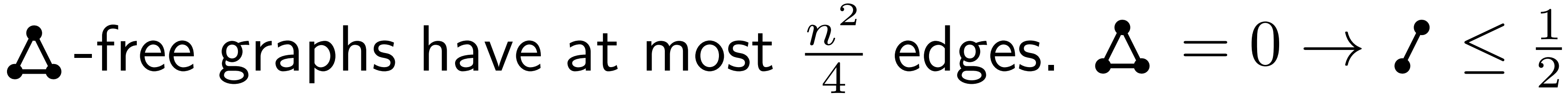
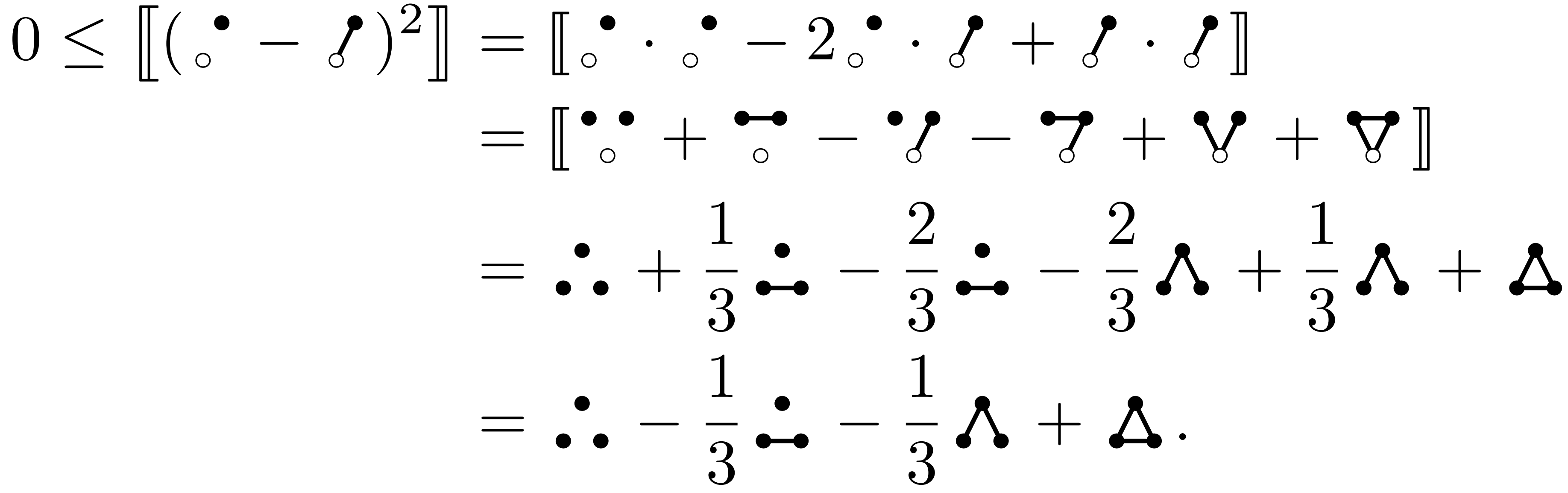
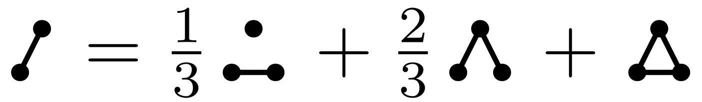


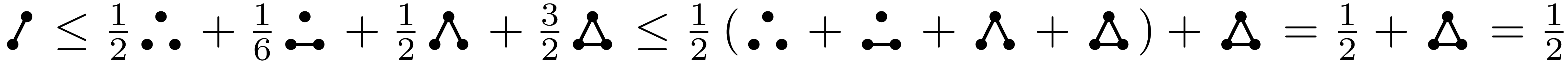


Mantel's theorem



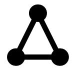






Balanced bipartitions of triangle-free graphs

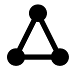
Balogh, Clemen, Lidický, 2022

-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

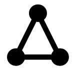
-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

 $\leq \frac{1}{2}$

Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

 -free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

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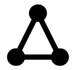
$[\![($  $- \frac{1}{3})^2]\!] \geq 10^{-4}$

Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

 $\leq \frac{1}{2}$

$[\![($  $- \frac{1}{3})^2]\!] \geq 10^{-4}$

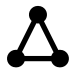
$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\frac{n}{2}$ vertices with at most $\frac{27n^2}{1024}$ edges

$$\frac{54n^2}{1024} + \varepsilon \leq \frac{n^2}{16}$$

Balanced bipartitions of triangle-free graphs

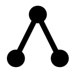
Balogh, Clemen, Lidický, 2022

-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

 $\leq \frac{1}{2}$

$[\![($  $- \frac{1}{3})^2]\!] \geq 10^{-4}$

$\frac{1}{3}$  $- \frac{2}{3}$  $+ \frac{1}{9} \geq 10^{-4}$

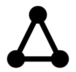
$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\frac{n}{2}$ vertices with at most $\frac{27n^2}{1024}$ edges

$$\frac{54n^2}{1024} + \varepsilon \leq \frac{n^2}{16}$$

Balanced bipartitions of triangle-free graphs

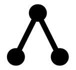
Balogh, Clemen, Lidický, 2022

-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

 $\leq \frac{1}{2}$

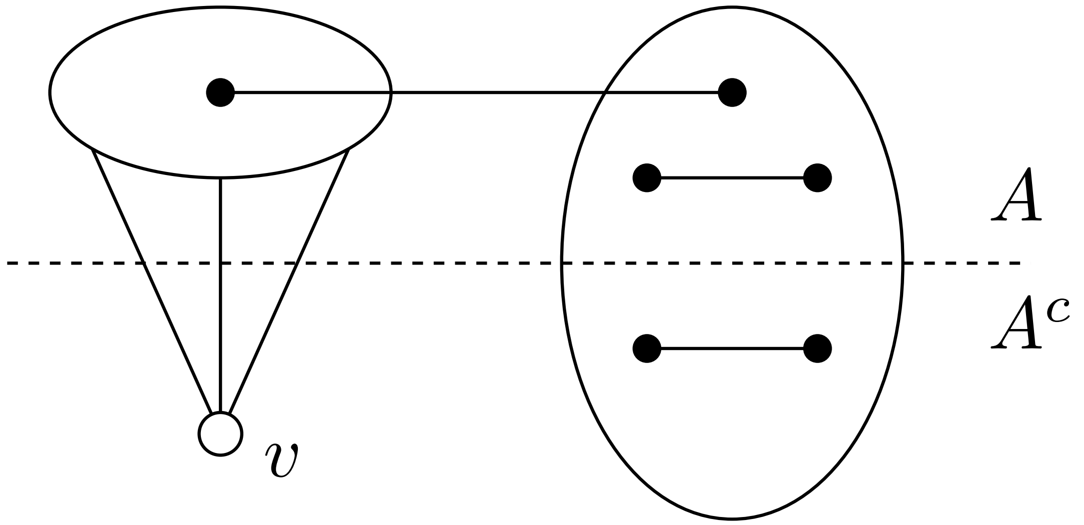
$\llbracket($  $- \frac{1}{3})^2\rrbracket \geq 10^{-4}$

$\frac{1}{3}$  $- \frac{2}{3}$  $+ \frac{1}{9} \geq 10^{-4}$

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\frac{n}{2}$ vertices with at most $\frac{27n^2}{1024}$ edges

$$\frac{54n^2}{1024} + \varepsilon \leq \frac{n^2}{16}$$

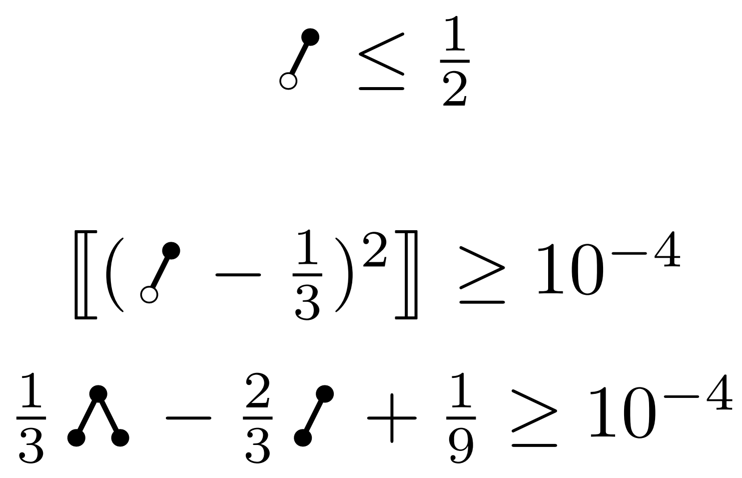


Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

△-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges



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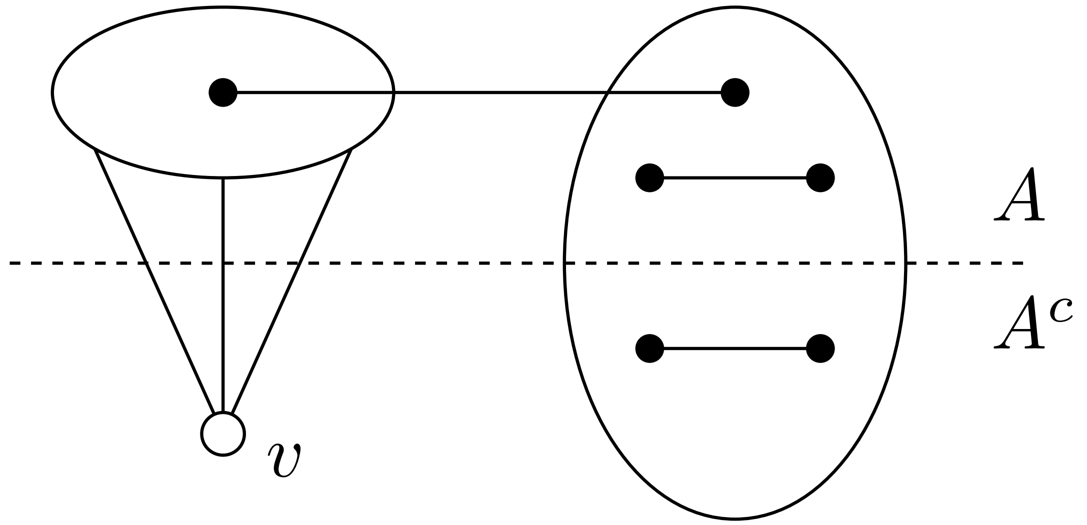

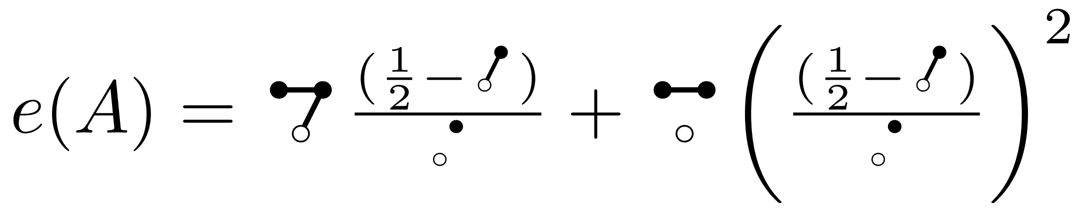

Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

$$\le \frac{1}{2}$$

$$[\![(\; - \tfrac{1}{3})^2]\!] \ge 10^{-4}$$

$$\tfrac{1}{3} \; - \tfrac{2}{3} \; + \tfrac{1}{9} \ge 10^{-4}$$

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

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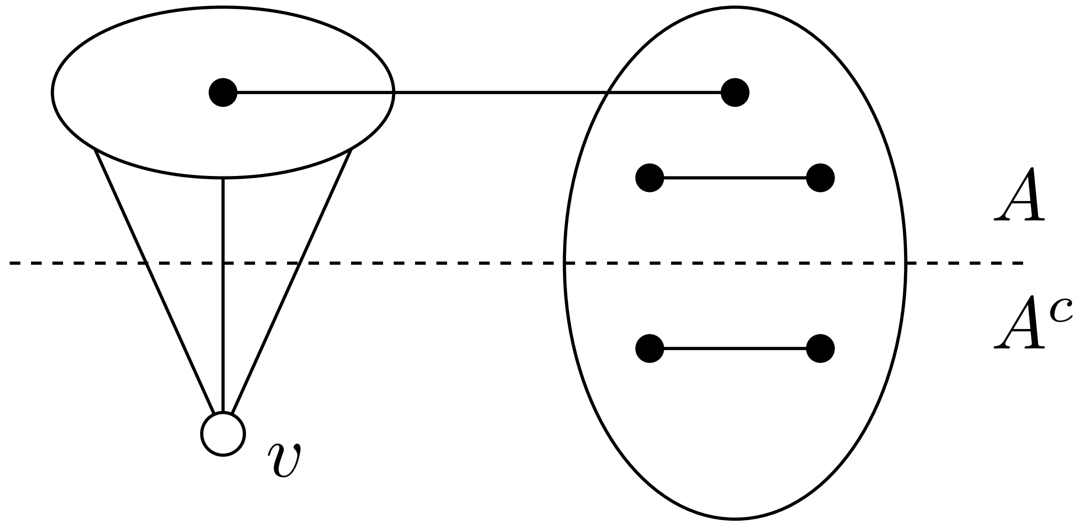

$$e(A) = \; \frac{(\frac{1}{2} - \;)}{\;} + \; \left(\frac{(\frac{1}{2} - \;)}{\;} \right)^2$$

$$e(A^c) = \; \left(\frac{\frac{1}{2}}{\;} \right)^2$$

Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

$\triangle$-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

$$\square \leq \frac{1}{2}$$

$$[\![(\square - \tfrac{1}{3})^2]\!] \geq 10^{-4}$$

$$\tfrac{1}{3}\square - \tfrac{2}{3}\square + \tfrac{1}{9} \geq 10^{-4}$$

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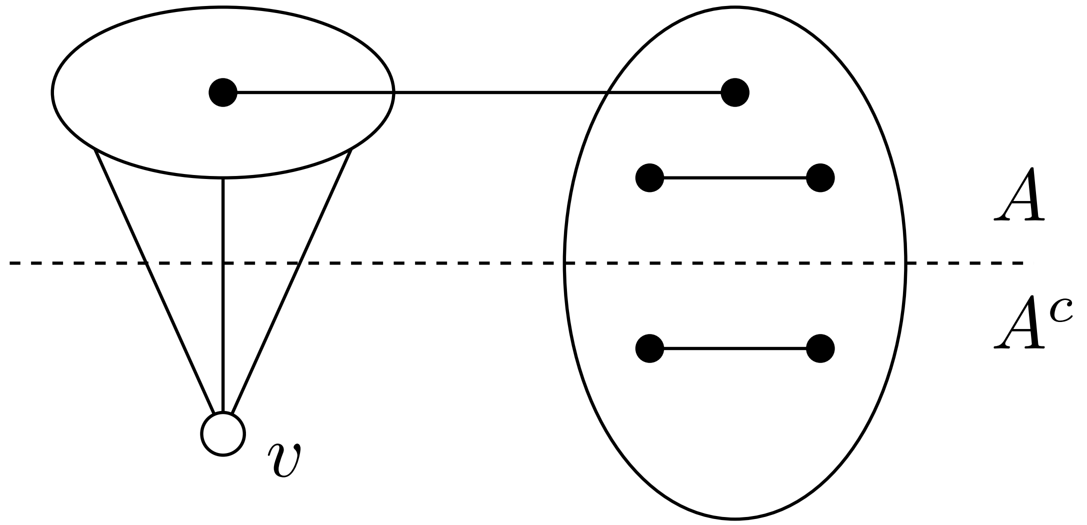


$$e(A) = \square \frac{(\frac{1}{2} - \square)}{\square} + \square \left(\frac{(\frac{1}{2} - \square)}{\square}\right)^2$$

$$e(A^c) = \square \left(\frac{\frac{1}{2}}{\square}\right)^2$$

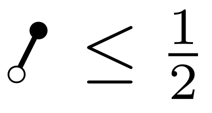
$$\frac{1}{8} \leq \square \frac{(\frac{1}{2} - \square)}{\square} + \square \left(\frac{(\frac{1}{2} - \square)}{\square}\right)^2 + \square \left(\frac{\frac{1}{2}}{\square}\right)^2$$

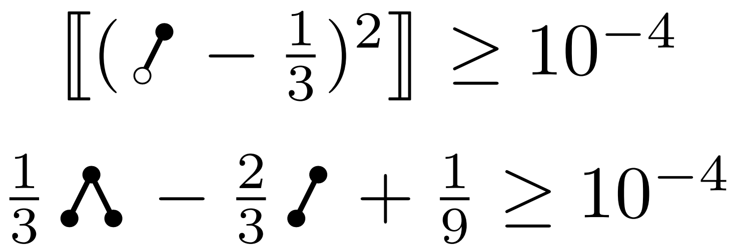
Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

△-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges





$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

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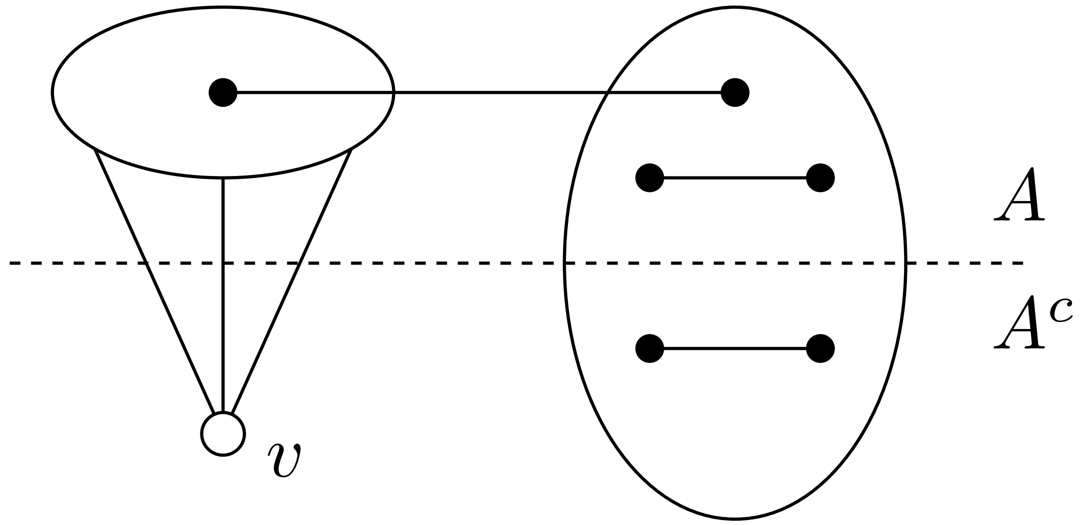

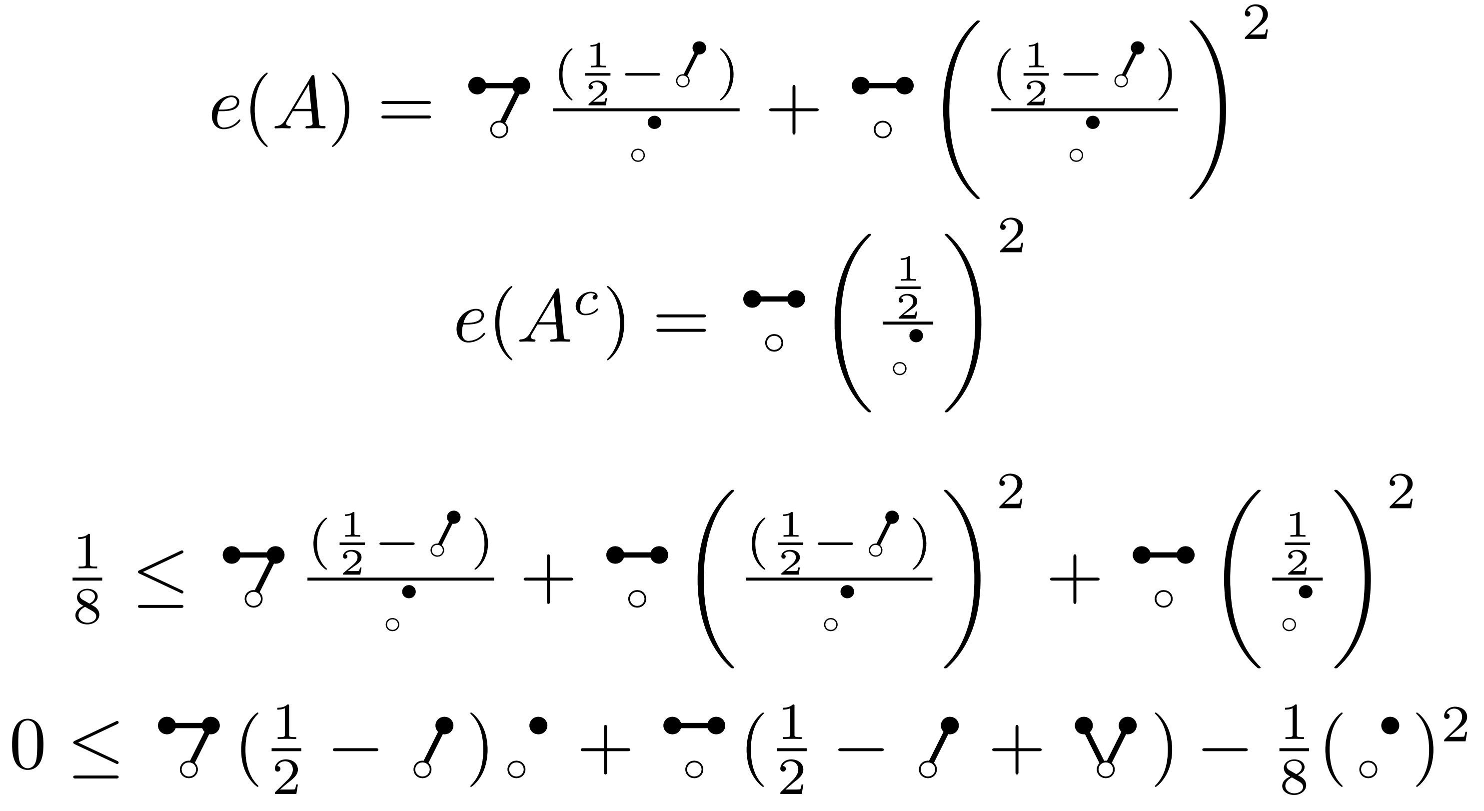

Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022

[triangle]-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges

$$[\text{rooted edge}] \le \frac{1}{2}$$

$$[\![([\text{rooted edge}] - \tfrac{1}{3})^2]\!] \ge 10^{-4}$$

$$\tfrac{1}{3}[\text{cherry}] - \tfrac{2}{3}[\text{edge}] + \tfrac{1}{9} \ge 10^{-4}$$

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

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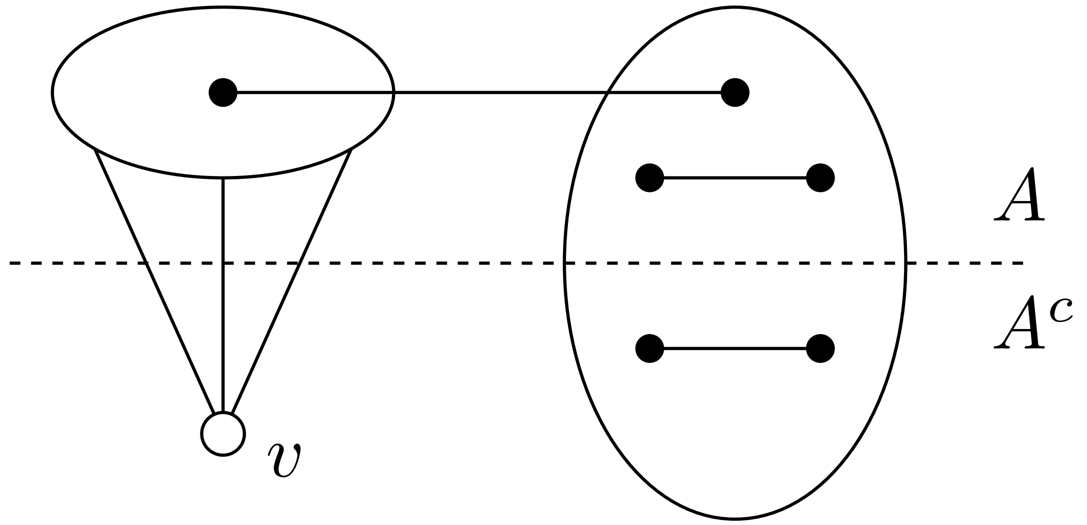
$$\frac{54n^2}{1024} + \varepsilon \le \frac{n^2}{16}$$



$$e(A) = [\text{flag}]\,\frac{(\frac12 - [\text{rooted edge}])}{[\text{rooted vertex}]} + [\text{flag}]\left(\frac{(\frac12 - [\text{rooted edge}])}{[\text{rooted vertex}]}\right)^2$$

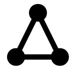
$$e(A^c) = [\text{flag}]\left(\frac{\frac12}{[\text{rooted vertex}]}\right)^2$$

$$\frac18 \le [\text{flag}]\,\frac{(\frac12 - [\text{rooted edge}])}{[\text{rooted vertex}]} + [\text{flag}]\left(\frac{(\frac12 - [\text{rooted edge}])}{[\text{rooted vertex}]}\right)^2 + [\text{flag}]\left(\frac{\frac12}{[\text{rooted vertex}]}\right)^2$$

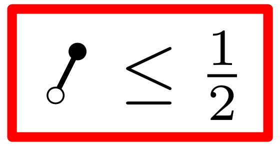
$$0 \le [\text{flag}]\,(\tfrac12 - [\text{rooted edge}])[\text{rooted vertex}] + [\text{flag}]\,(\tfrac12 - [\text{rooted edge}] + [\text{rooted cherry}]) - \tfrac18([\text{rooted vertex}])^2$$

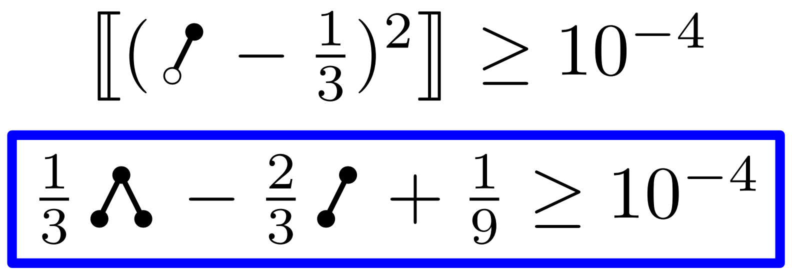
Balanced bipartitions of triangle-free graphs

Balogh, Clemen, Lidický, 2022


-free graphs

can be made balanced bipartite by removing at most $\frac{n^2}{16}$ edges





$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

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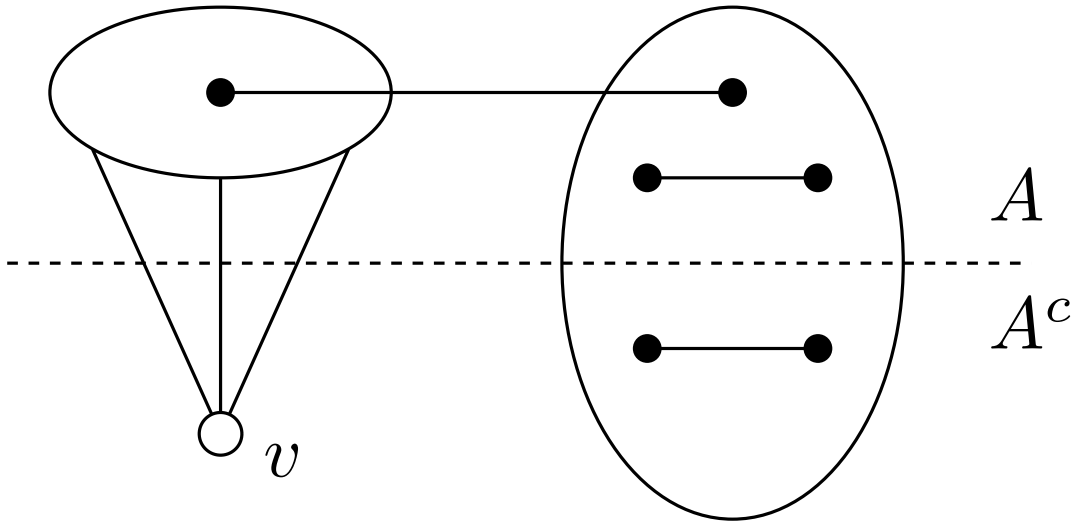
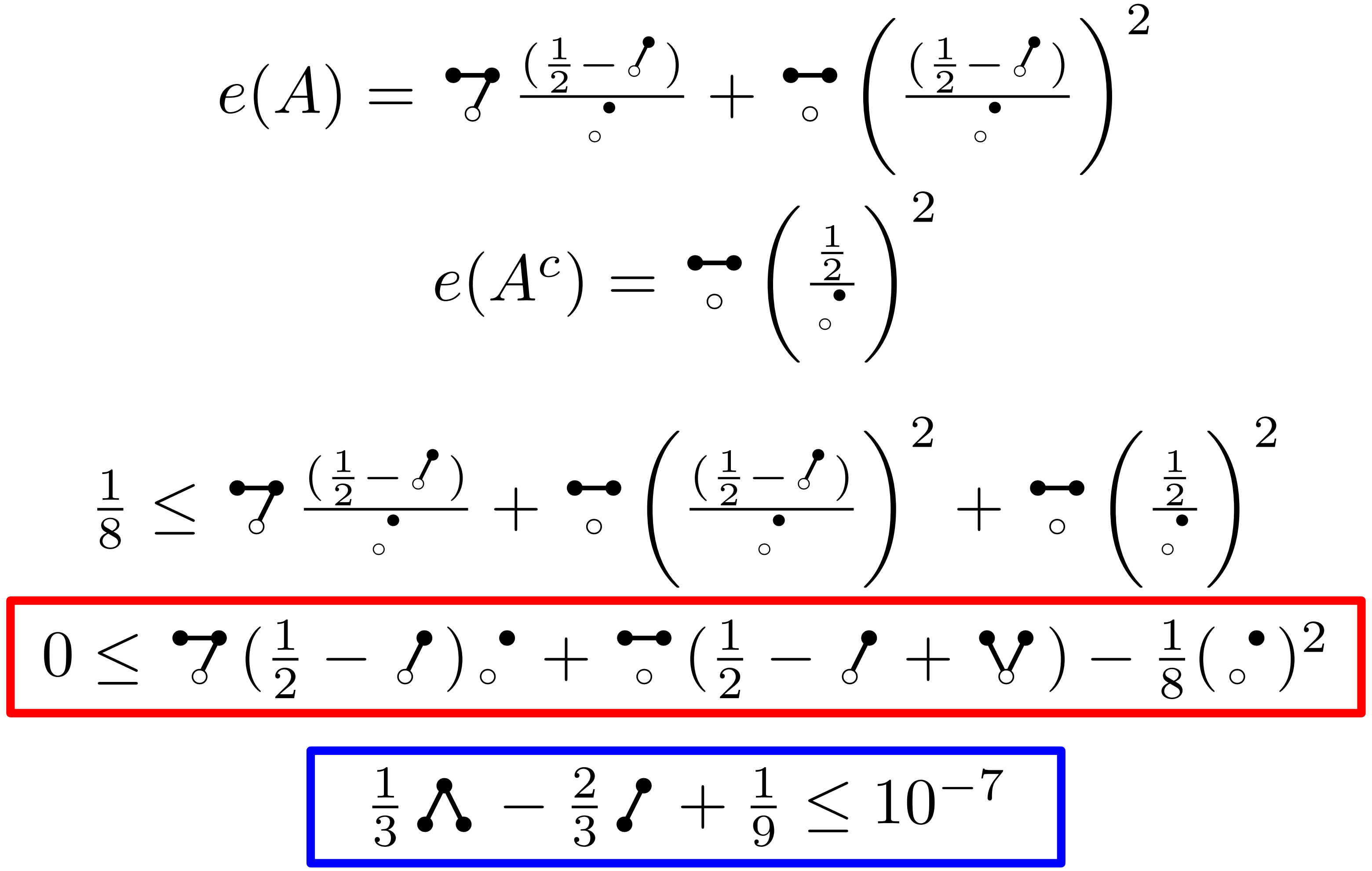
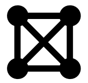
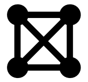




 -free

	Sparse half	Make bipartite	Make balanced bipartite
 -free	$\frac{n^2}{25} \cdot \frac{1}{2}$	$\frac{n^2}{25}$	$\frac{n^2}{16}$
 -free	$\frac{n^2}{9} \cdot \frac{1}{2}$	$\frac{n^2}{9}$	$\frac{n^2}{9}$

 -free

	Sparse half	Make bipartite	Make balanced bipartite
 -free	$\frac{n^2}{25} \cdot \frac{1}{2}$	$\frac{n^2}{25}$	$\frac{n^2}{16}$
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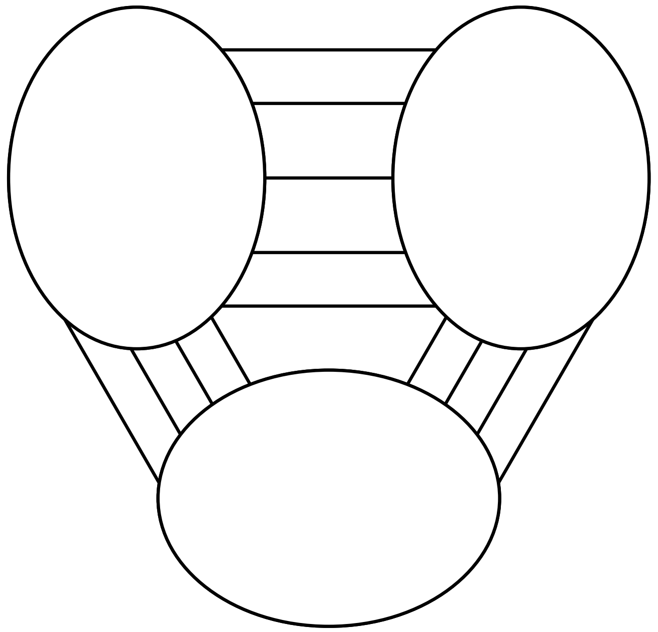


Balanced bipartitions of K_4-free graphs

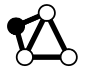
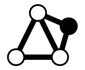
-free and tripartite - 2022

Balanced bipartitions of K_4-free graphs

-free and tripartite - 2022

-free and has two disjoint, independent sets I_1, I_2, so that $|I_1| + |I_2| \geq \frac{2n}{3}$.

Balanced bipartitions of K_4-free graphs

- -free and tripartite - 2022
- -free and has two disjoint, independent sets I_1, I_2, so that $|I_1| + |I_2| \geq \frac{2n}{3}$.  $+$  $\leq \frac{2}{3}$

Balanced bipartitions of K_4-free graphs

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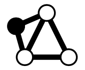
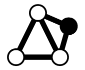
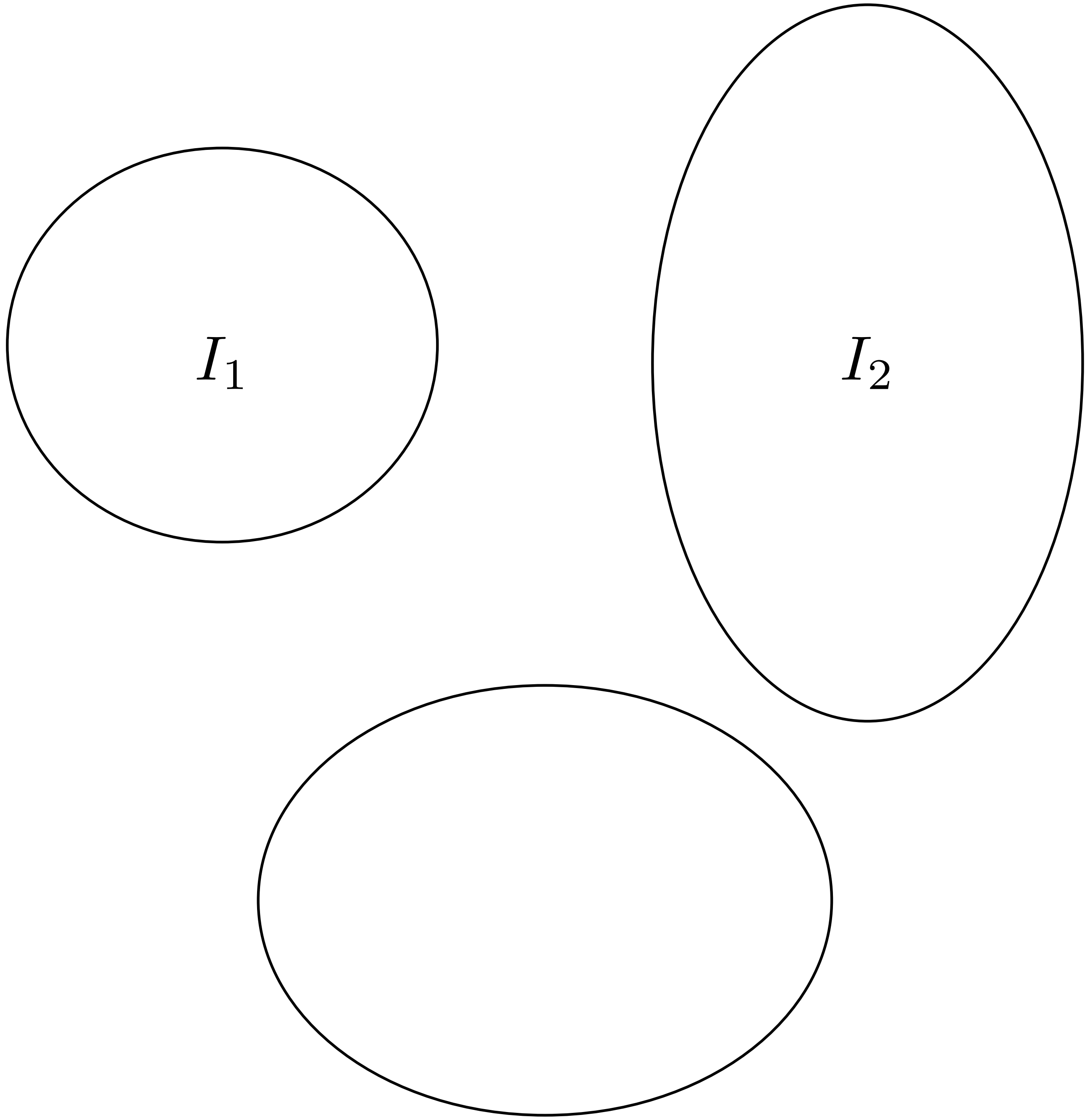
-free and has two disjoint, independent sets I_1, I_2, so that $|I_1| + |I_2| \geq \frac{2n}{3}$.  $+$  $\leq \frac{2}{3}$



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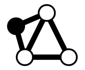
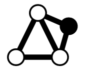
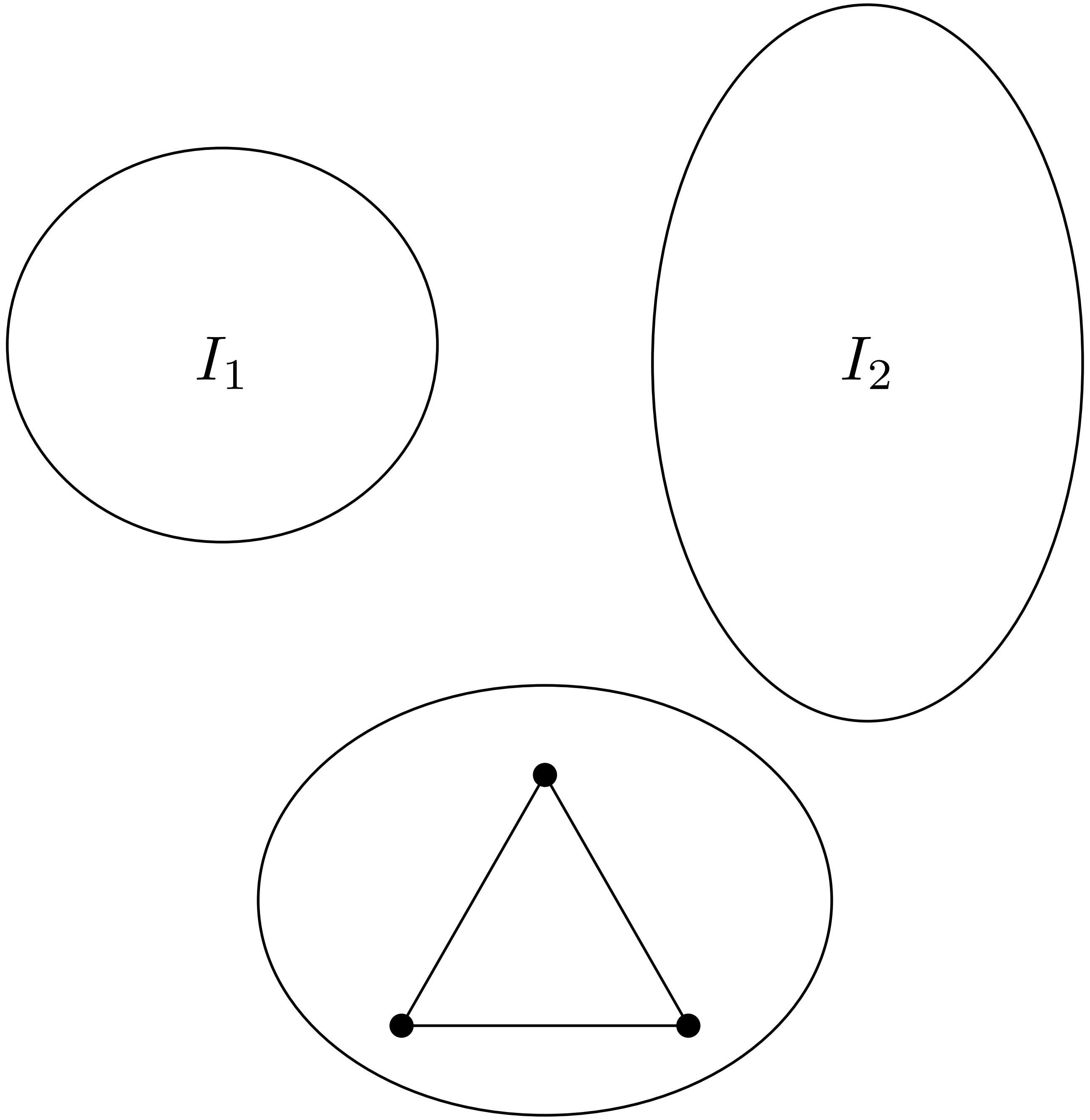
-free and has two disjoint, independent sets I_1, I_2, so that $|I_1| + |I_2| \geq \frac{2n}{3}$.  $+$  $\leq \frac{2}{3}$



Balanced bipartitions of K_4-free graphs

-free and tripartite - 2022

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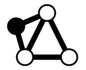
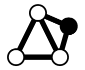
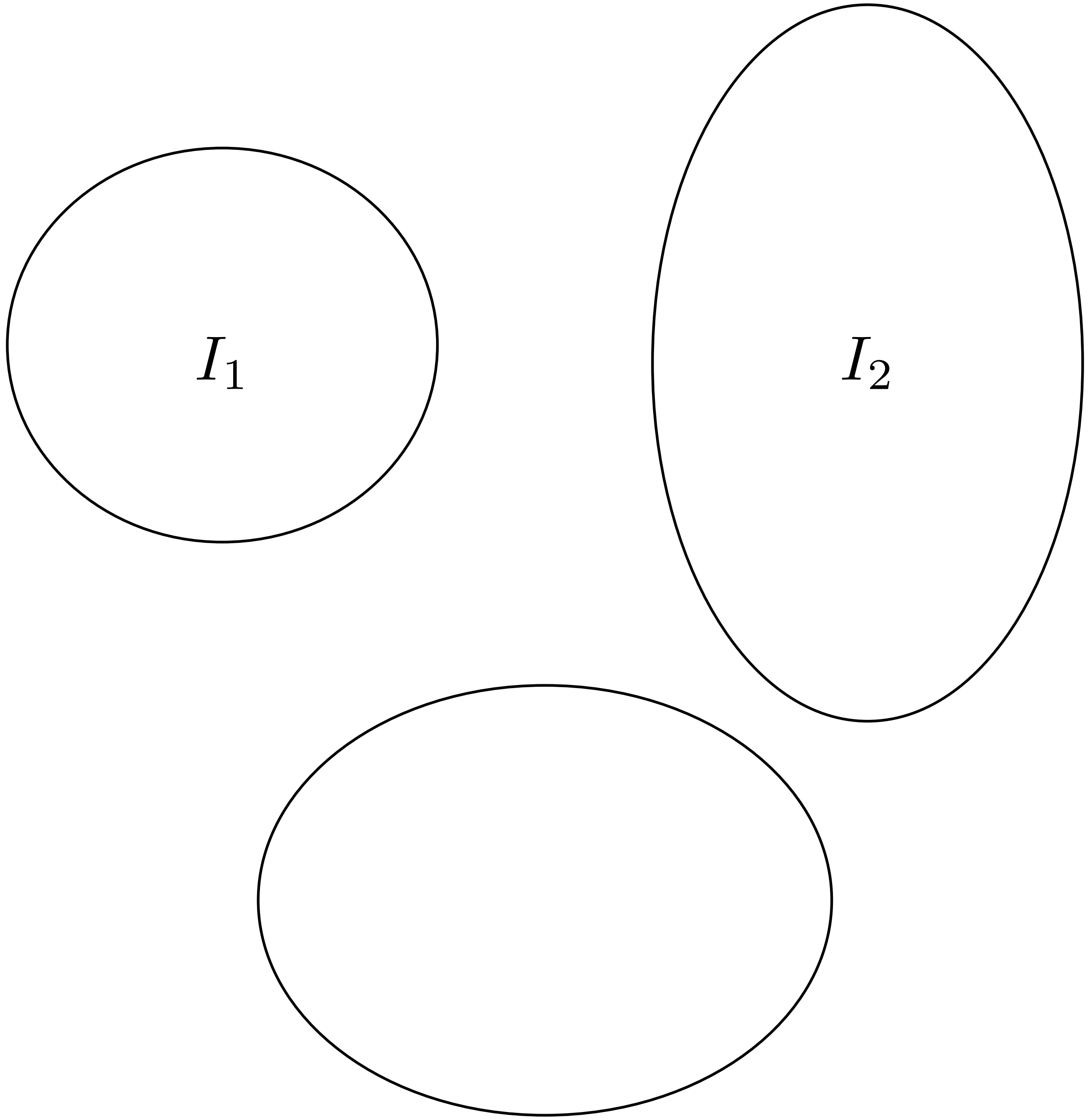
 $+$  $\leq \frac{2}{3}$



$[\![($ $- \frac{1}{3})^2]\!] \geq 10^{-4}$

$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

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Balanced bipartitions of K_4-free graphs

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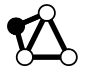
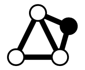
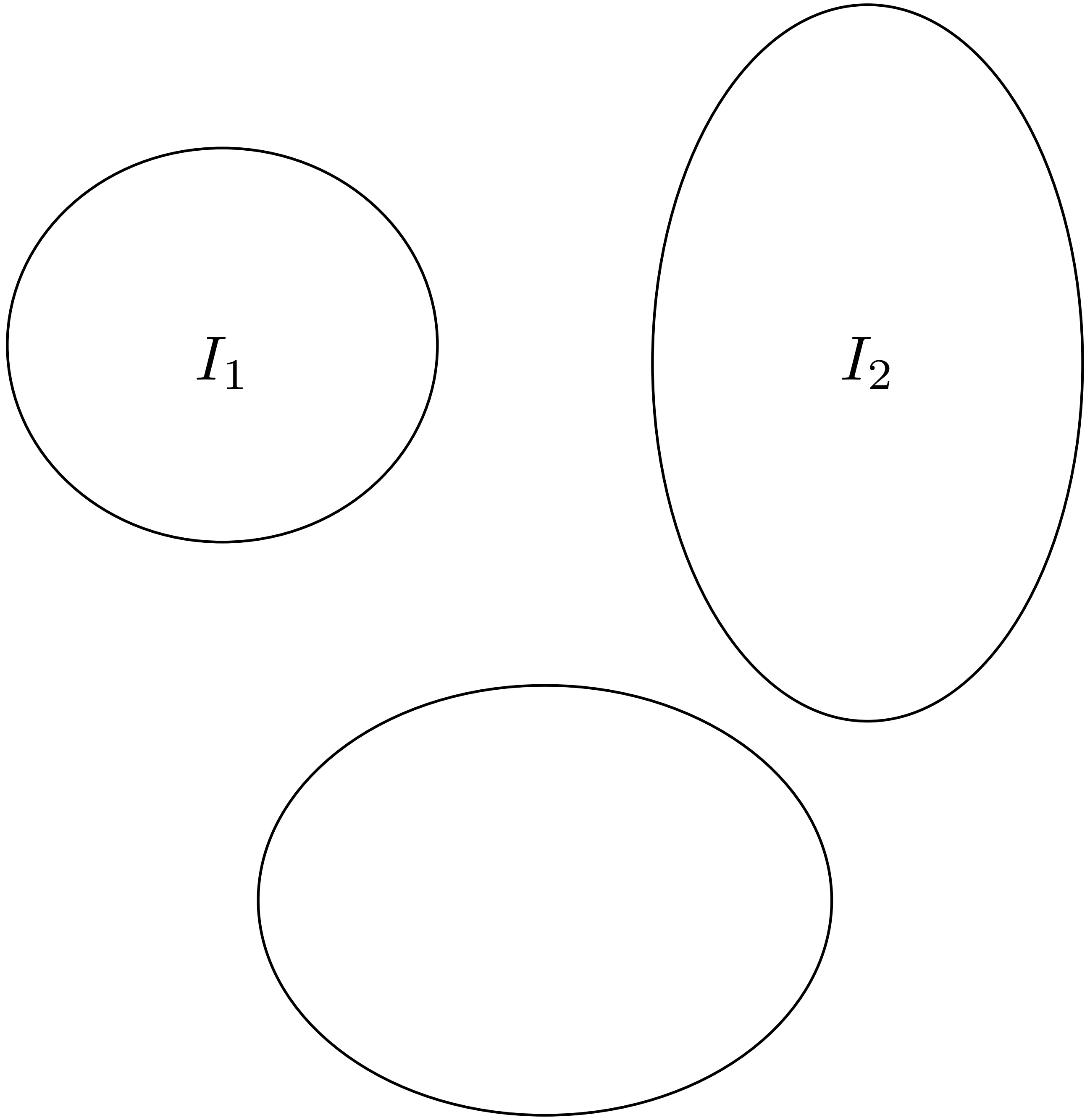
 $+$  $\leq \frac{2}{3}$



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$\frac{n}{2}$ vertices with at most $\frac{n^2}{50}$ edges?

$\frac{n}{2}$ vertices with at most $\frac{27n^2}{1024}$ edges

$$\frac{54n^2}{1024} + \varepsilon \leq \frac{n^2}{16}$$

$\frac{n}{2}$ vertices with at most $\frac{n^2}{18}$ edges

$$\frac{n^2}{9} + \varepsilon \not\leq \frac{n^2}{9}$$

References

1. Balogh, Clemen, Lidický, 2022 - 10 Problems for Partitions of Triangle-free Graphs.

2. Andrzej Grzesik - Extremal Graph Theory lecture notes.