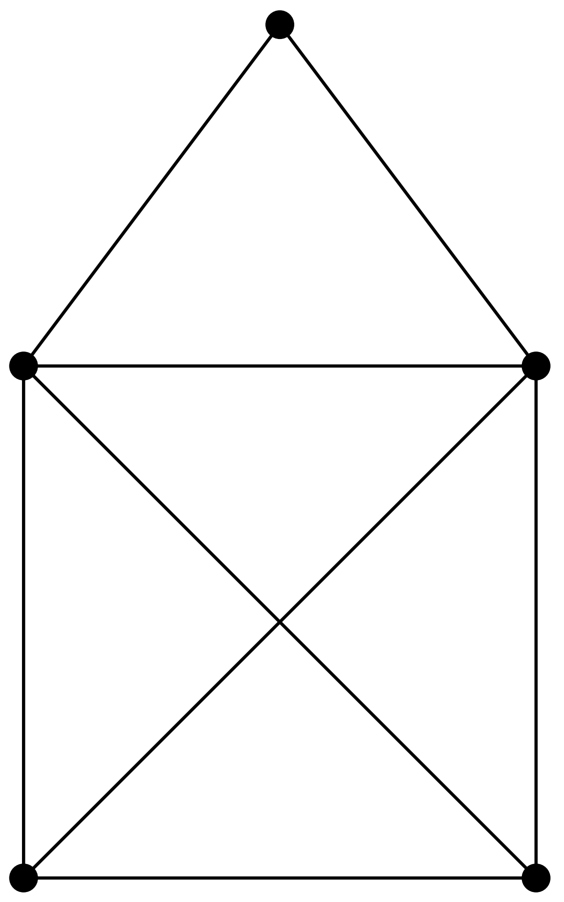
(Some of) the many uses of Eulerian graphs in graph theory (plus some applications)

based on an article by Herbert Fleischner

Bartłomiej Błoniarz

Eulerian graphs

Eulerian trail



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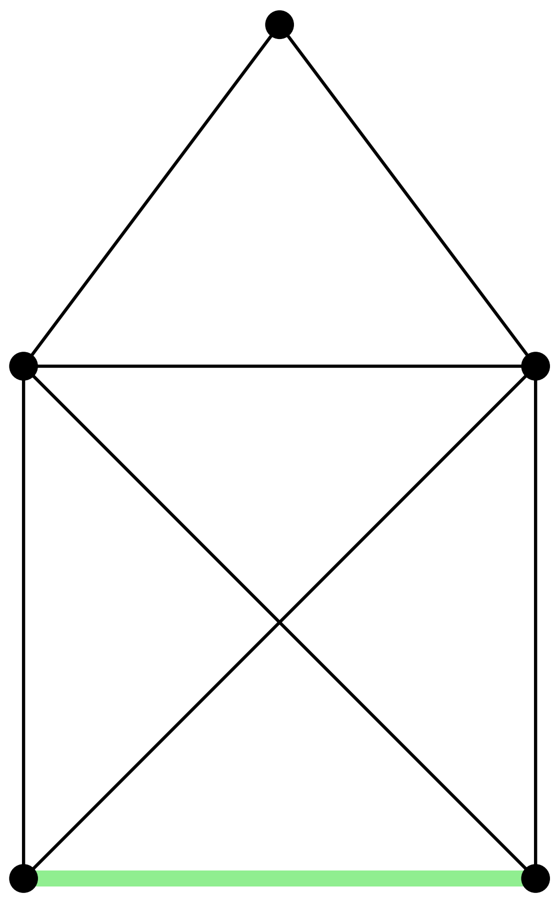

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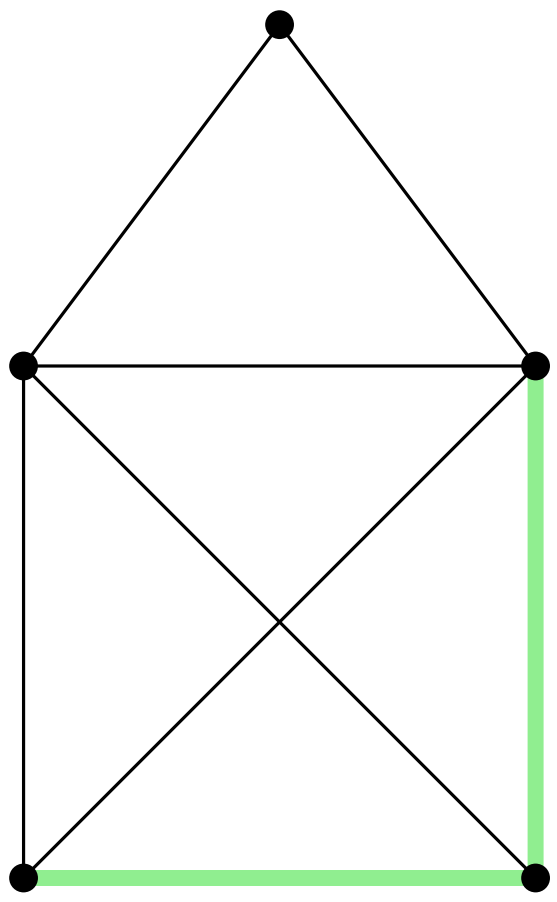

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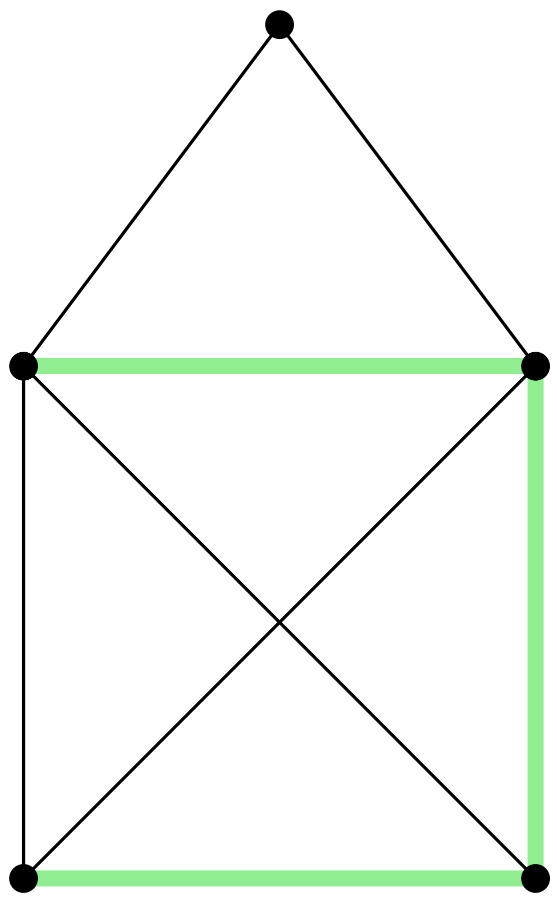

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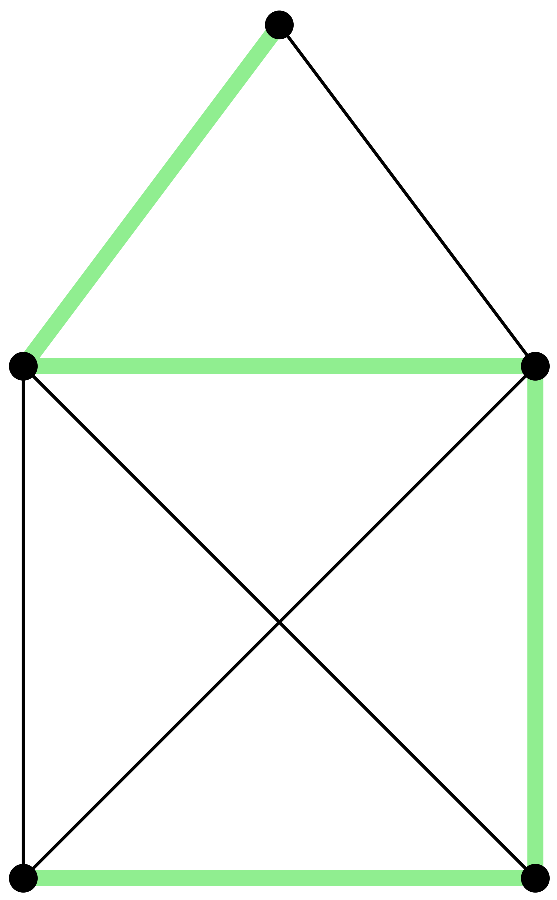

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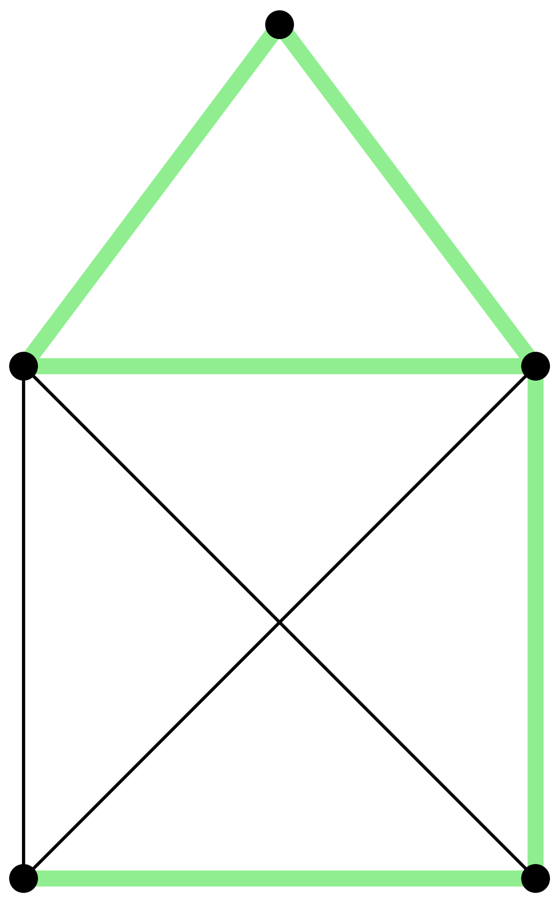

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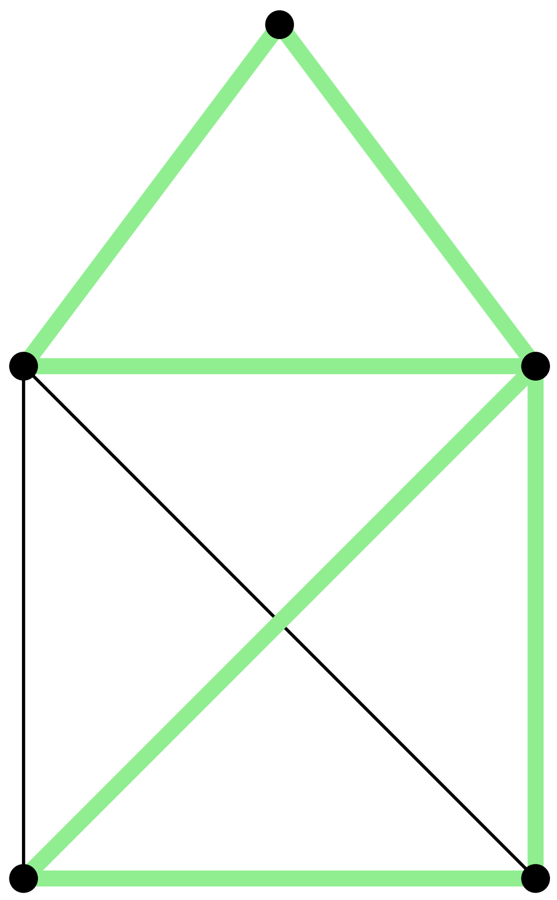

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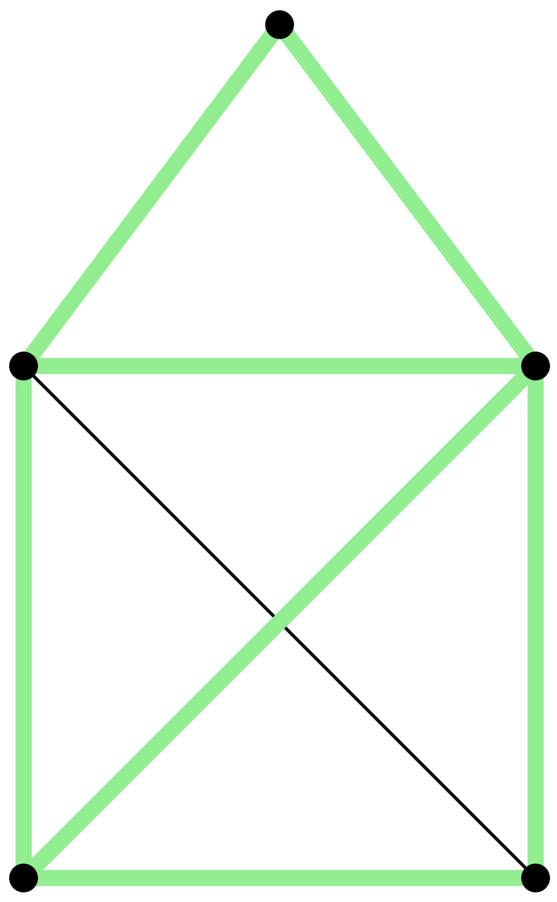

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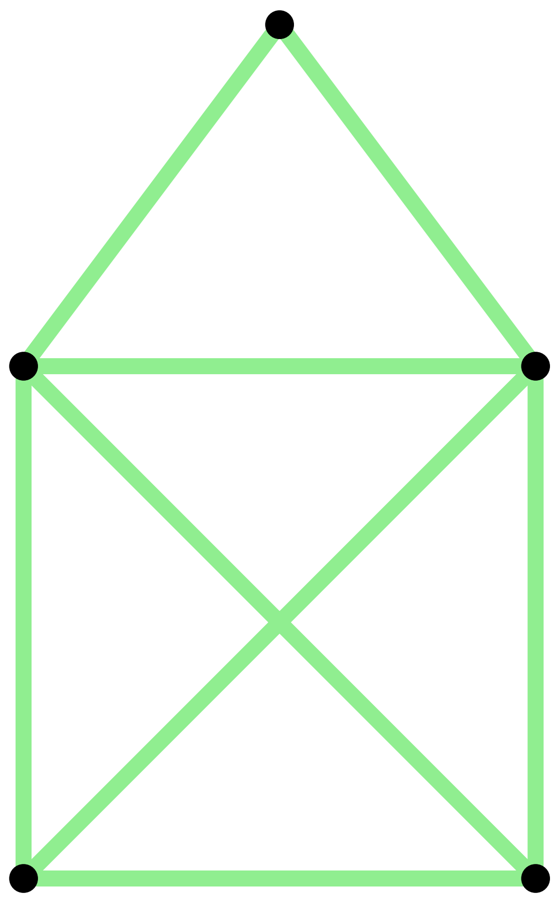

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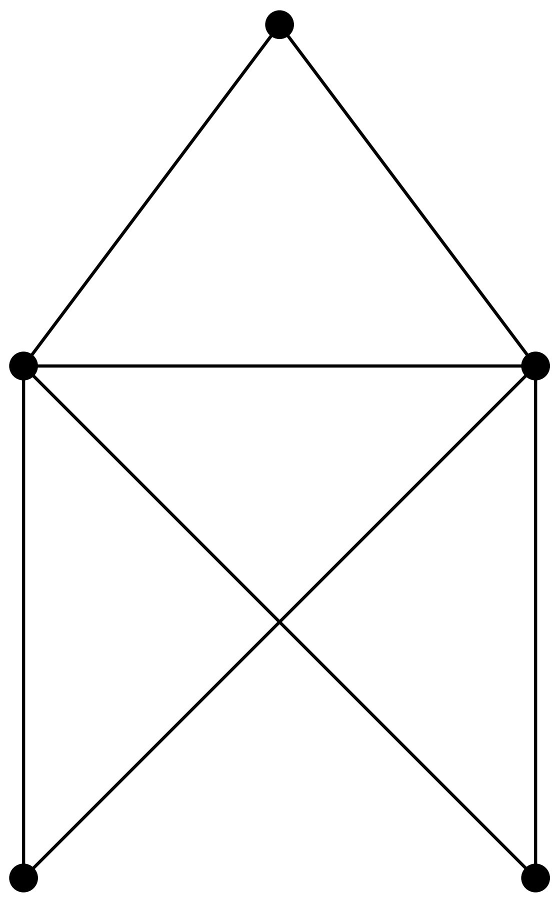

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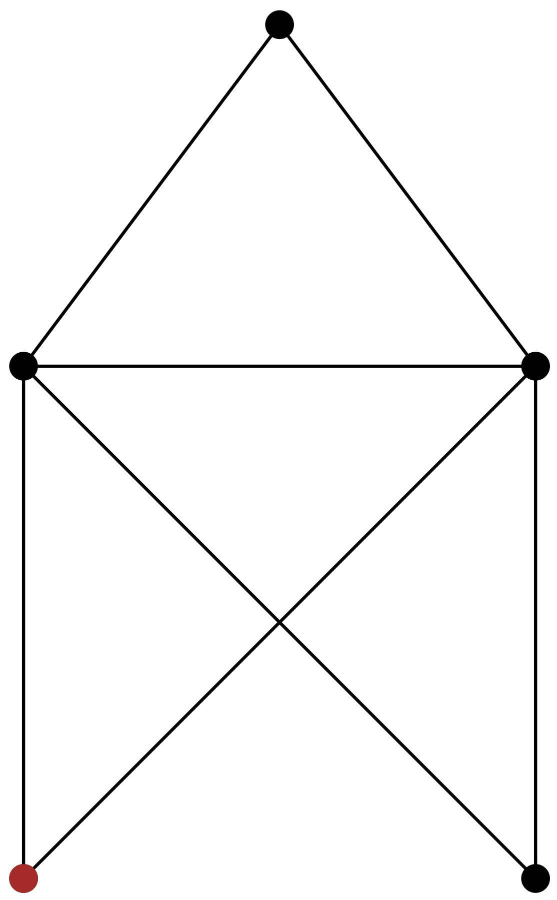

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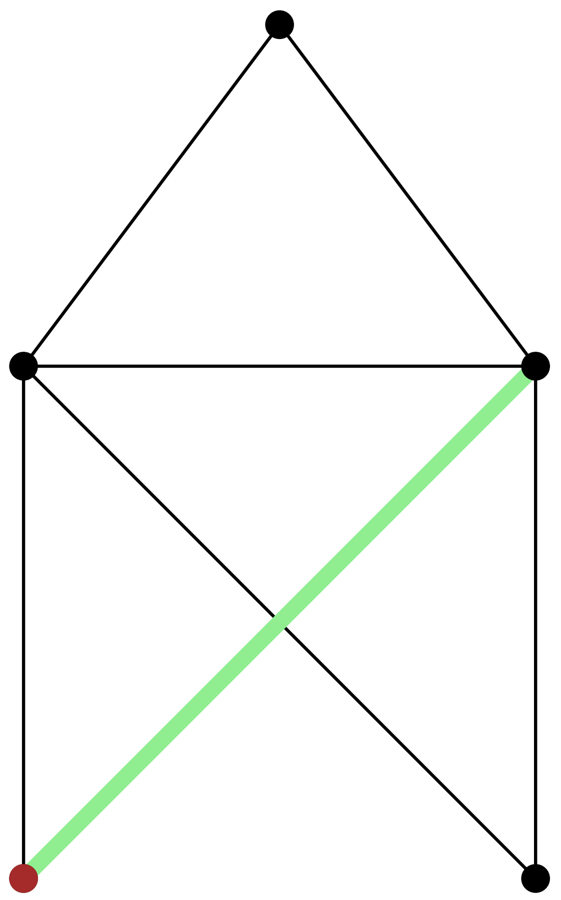

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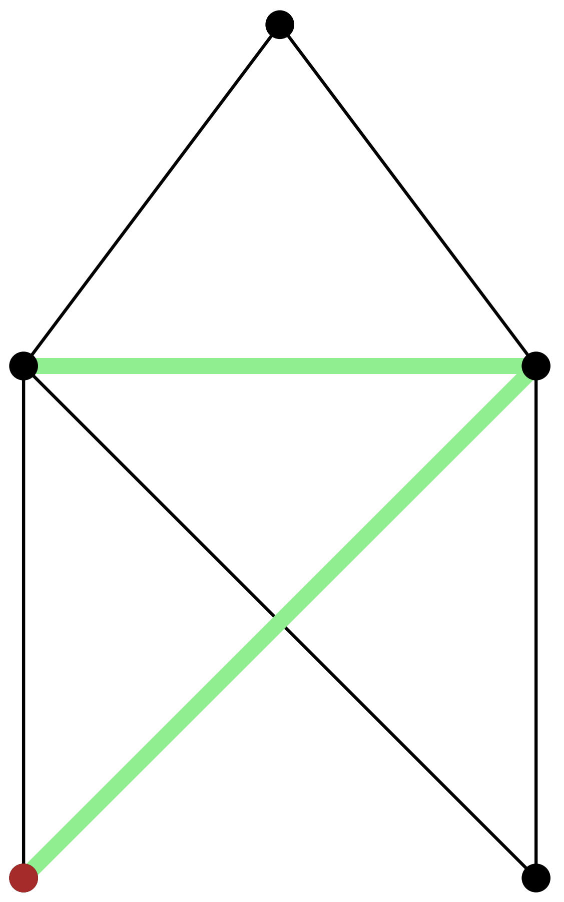

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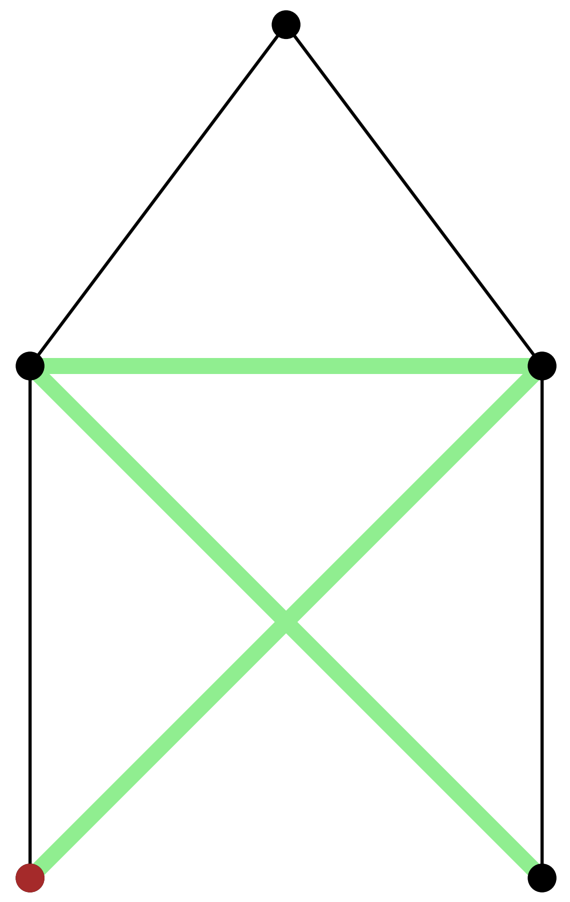

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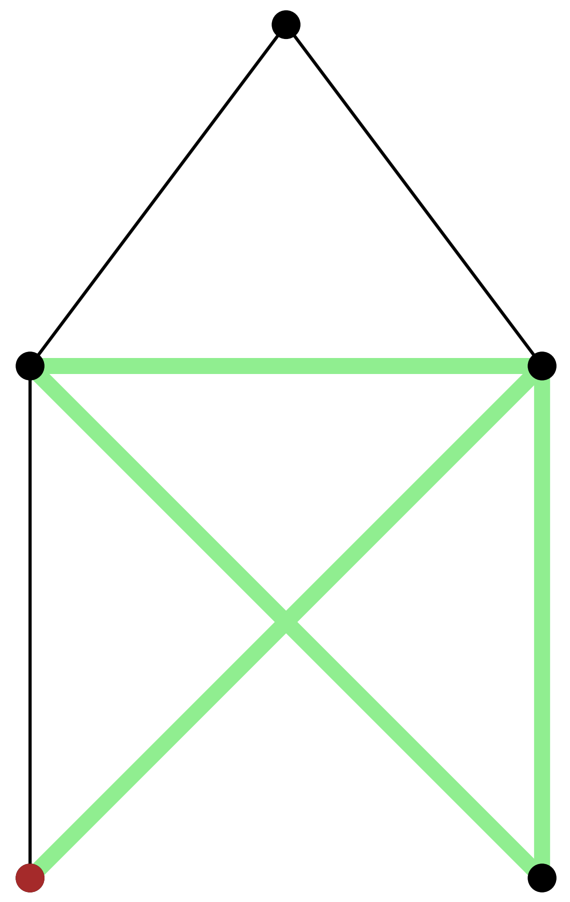

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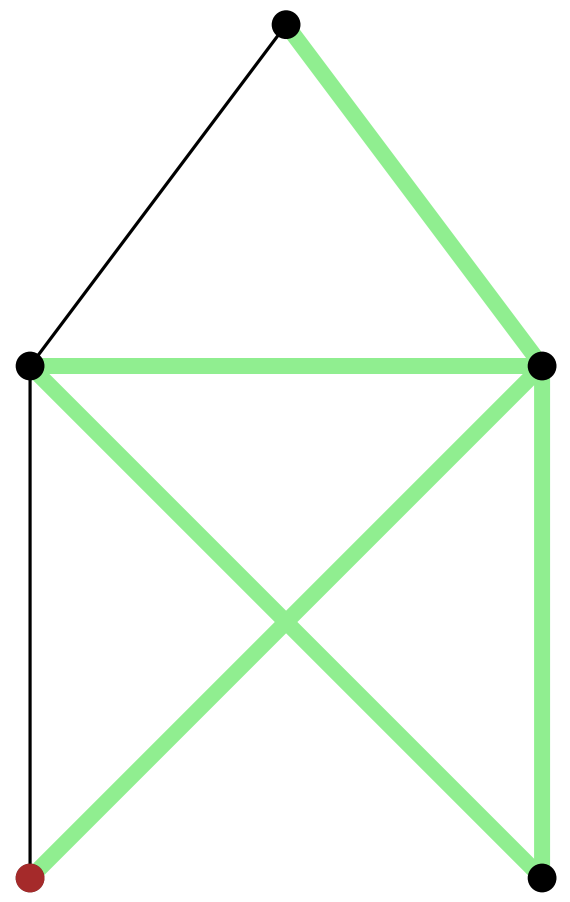

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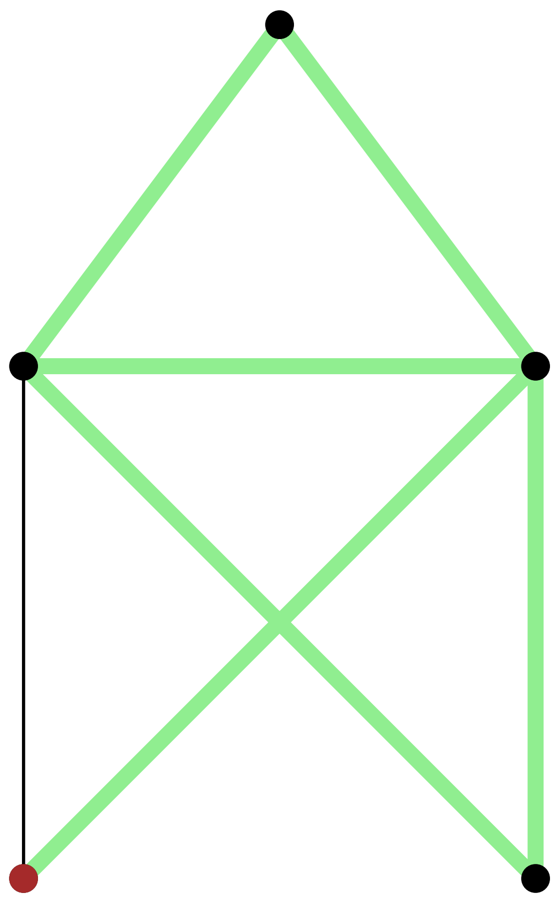

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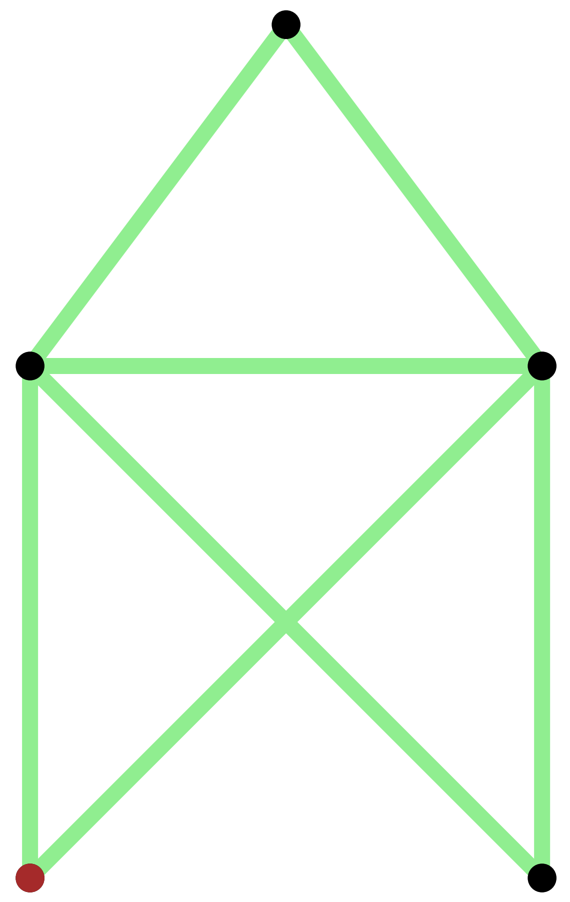

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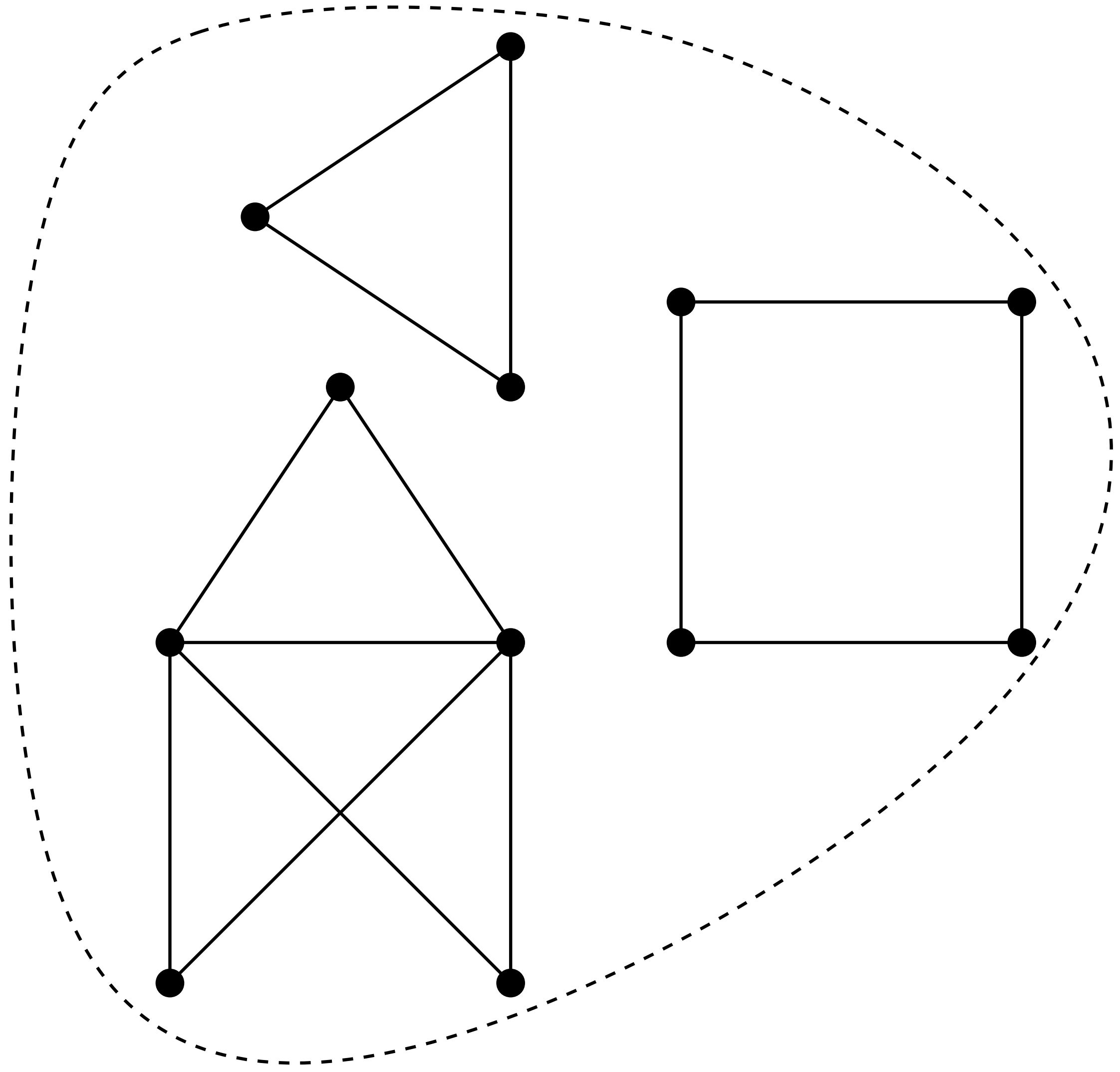
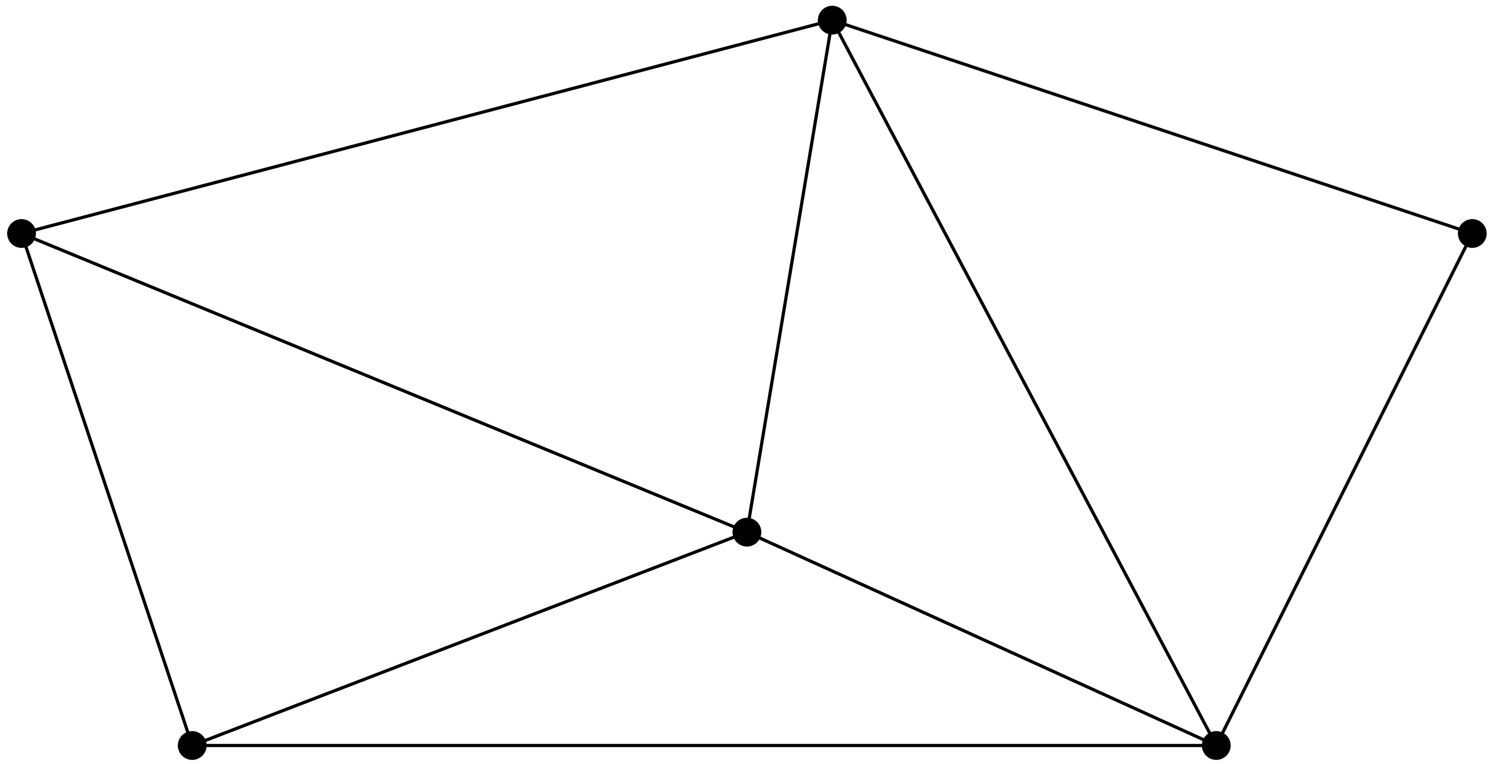


Euler-Hamiltonian connection

Hamiltonian graph - a graph with a cycle that visits each vertex exactly once



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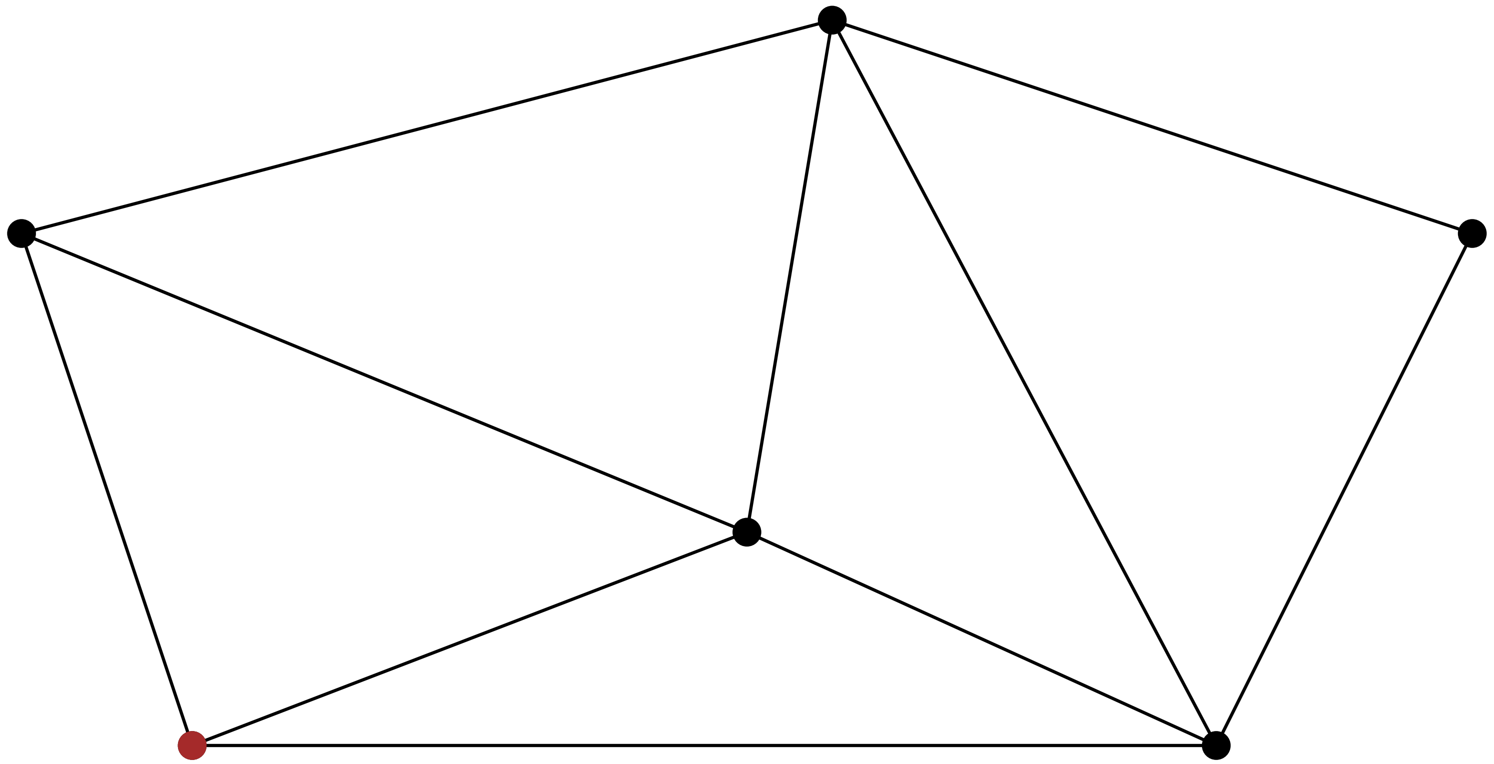

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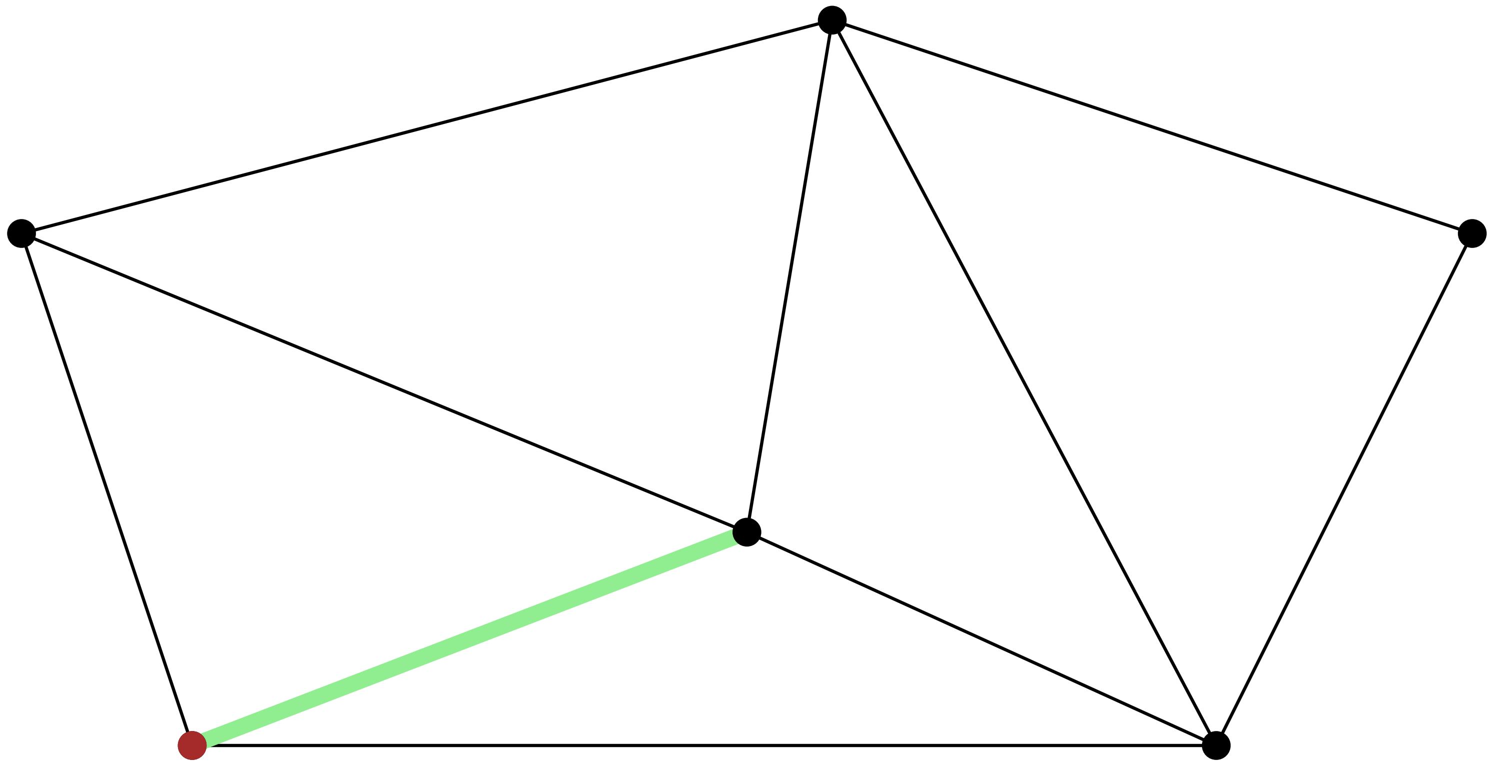

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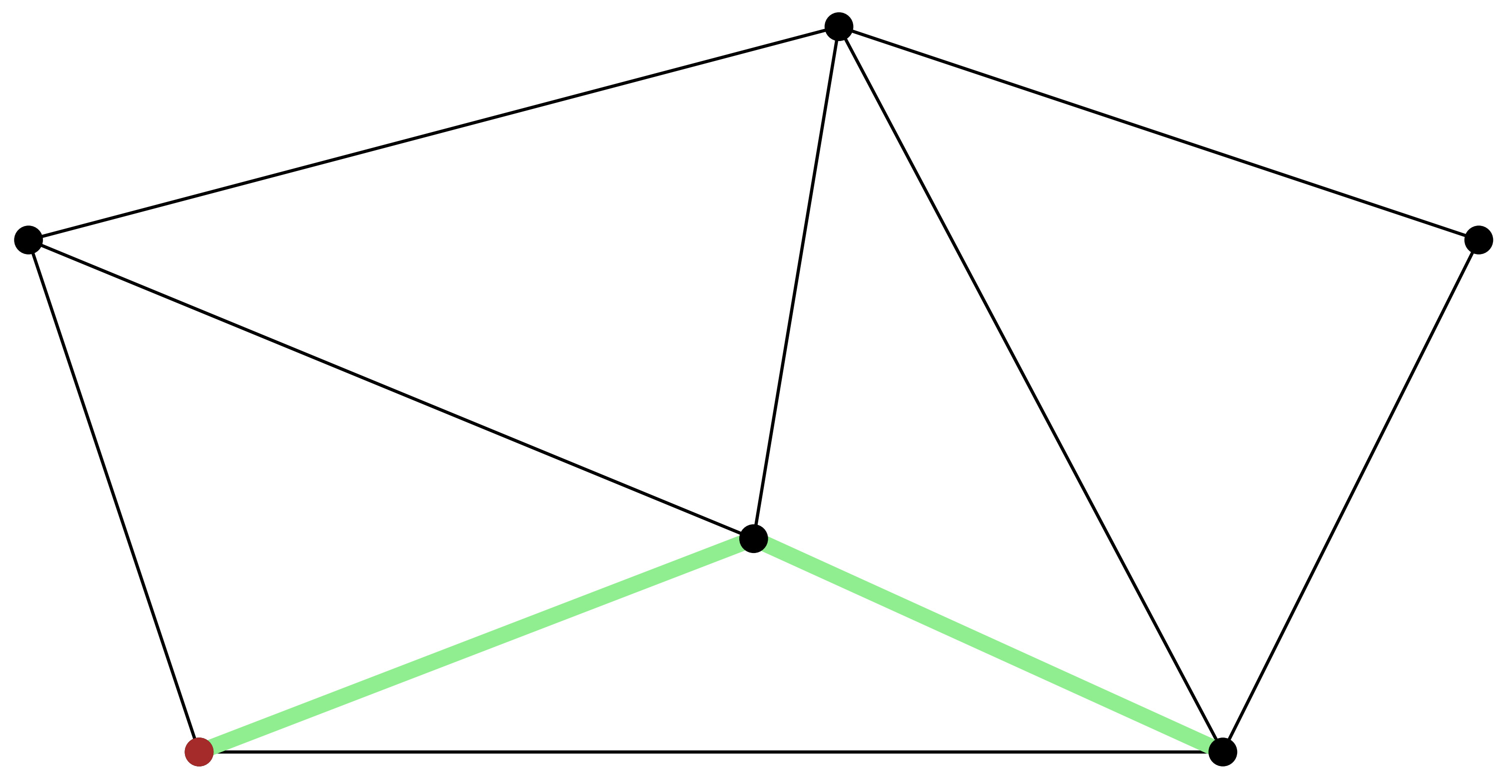

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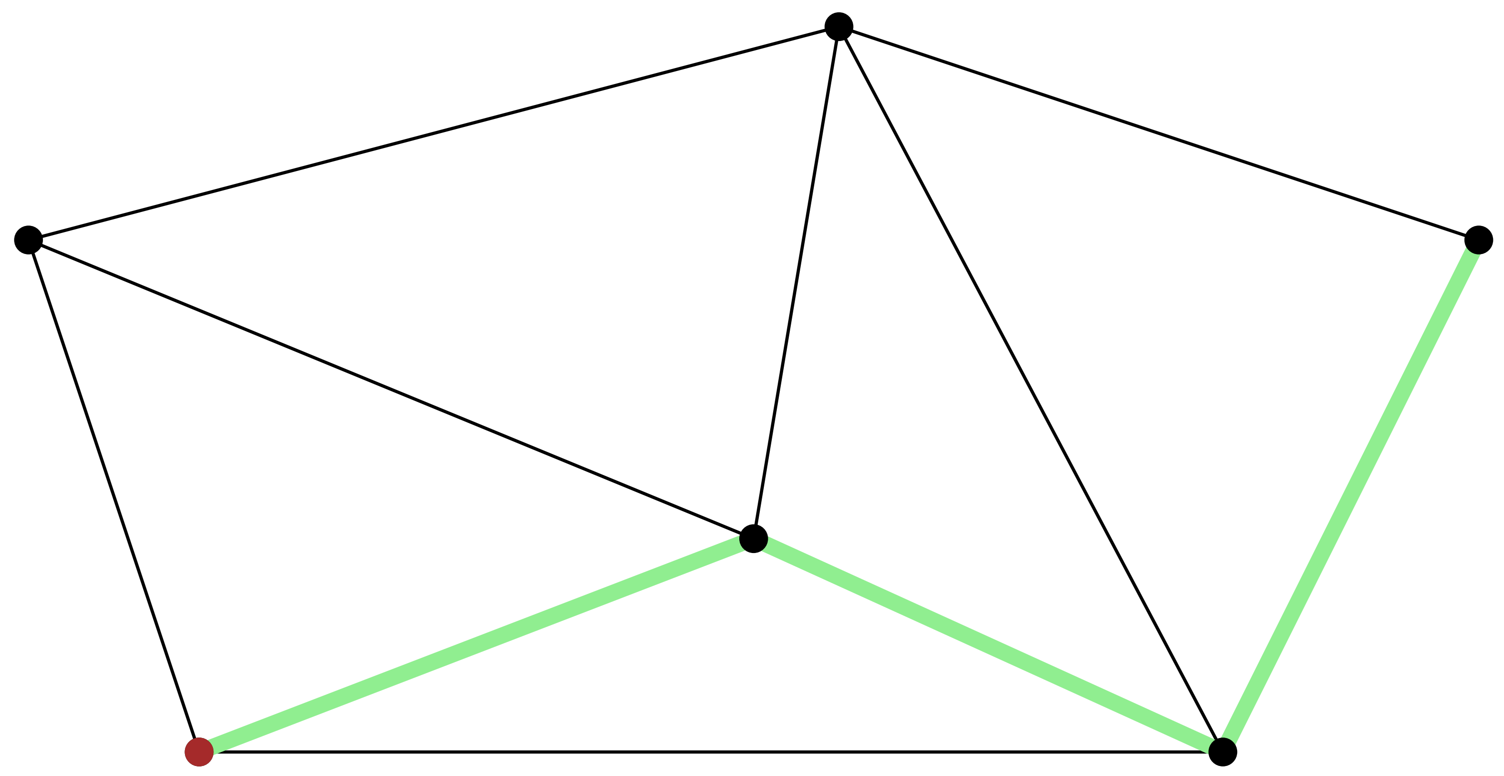

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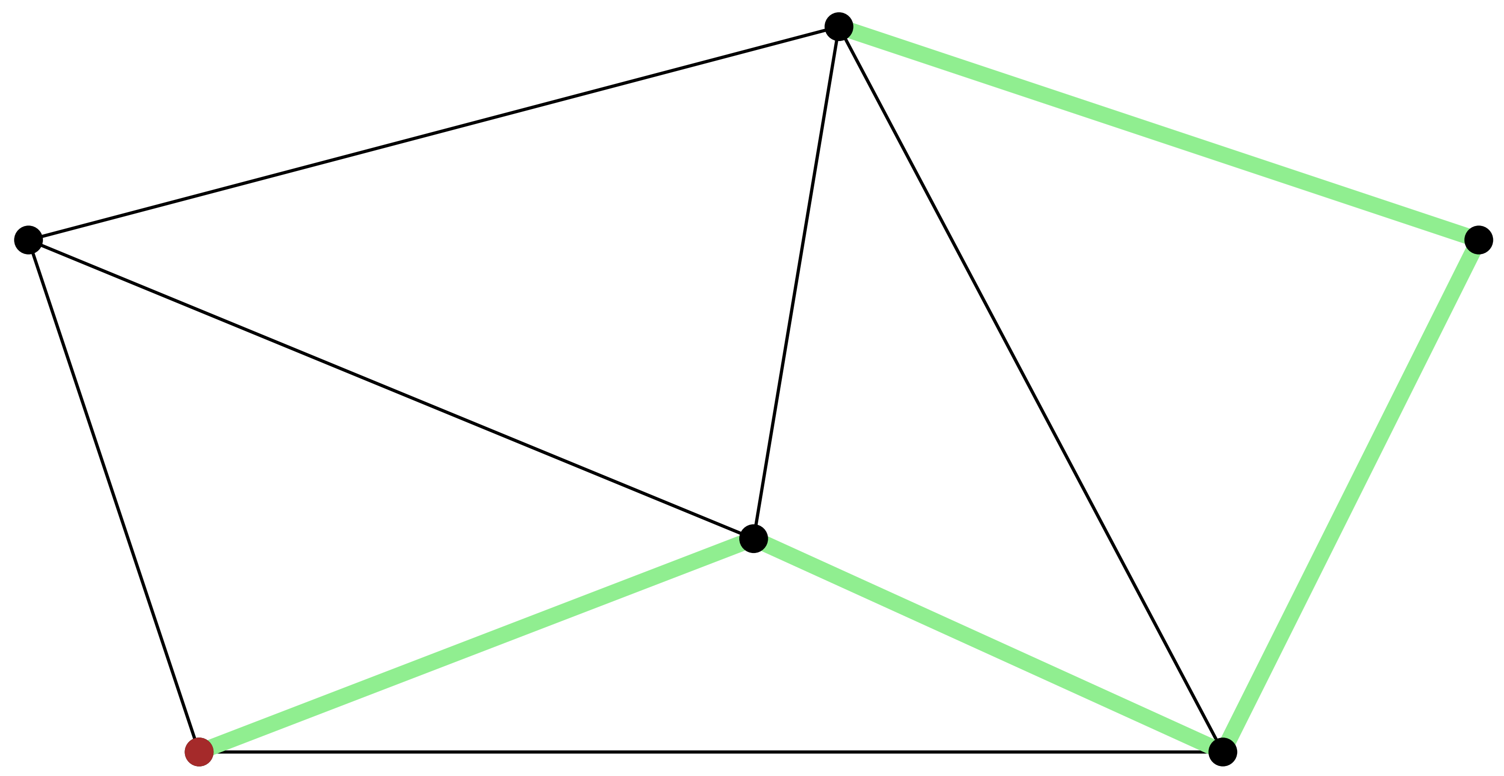

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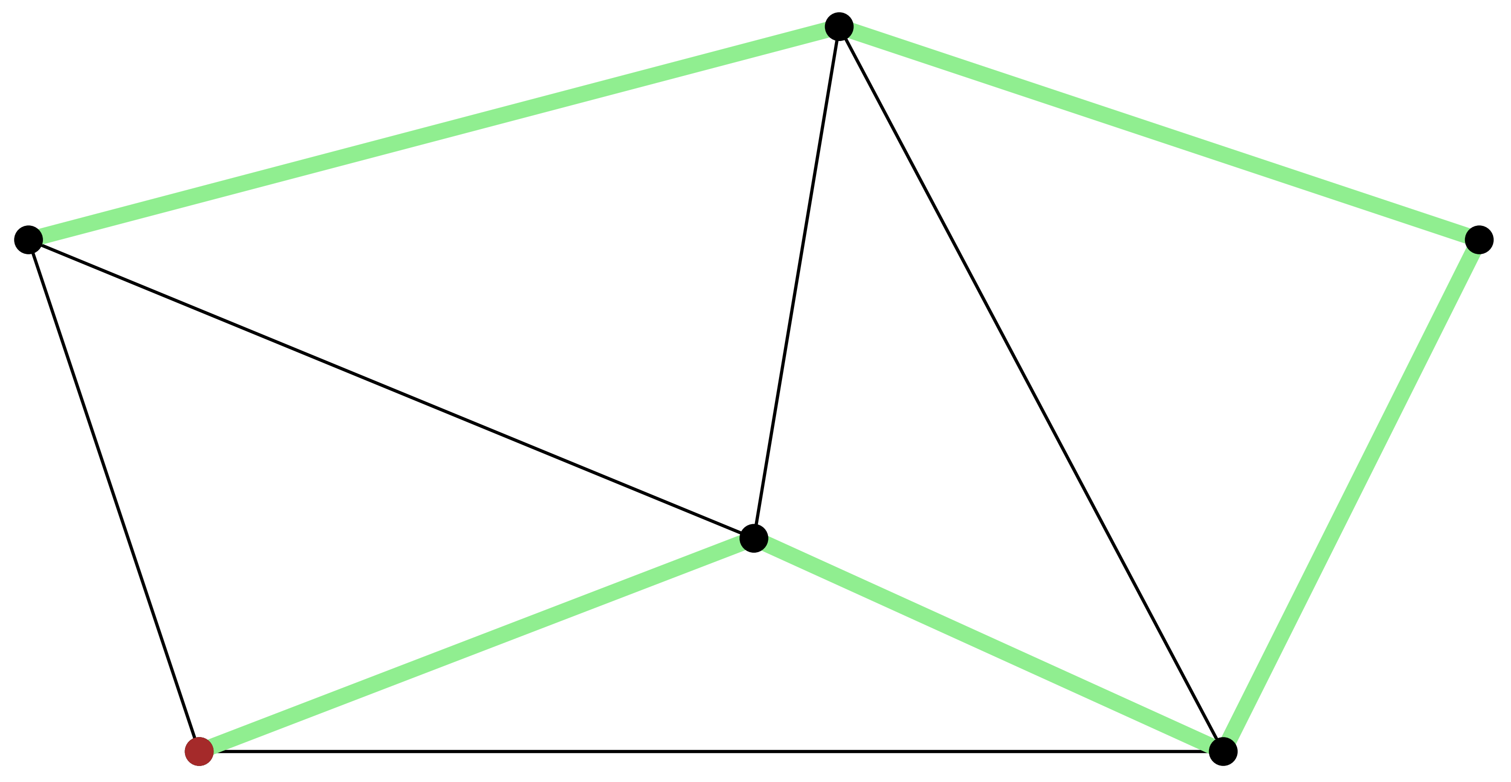

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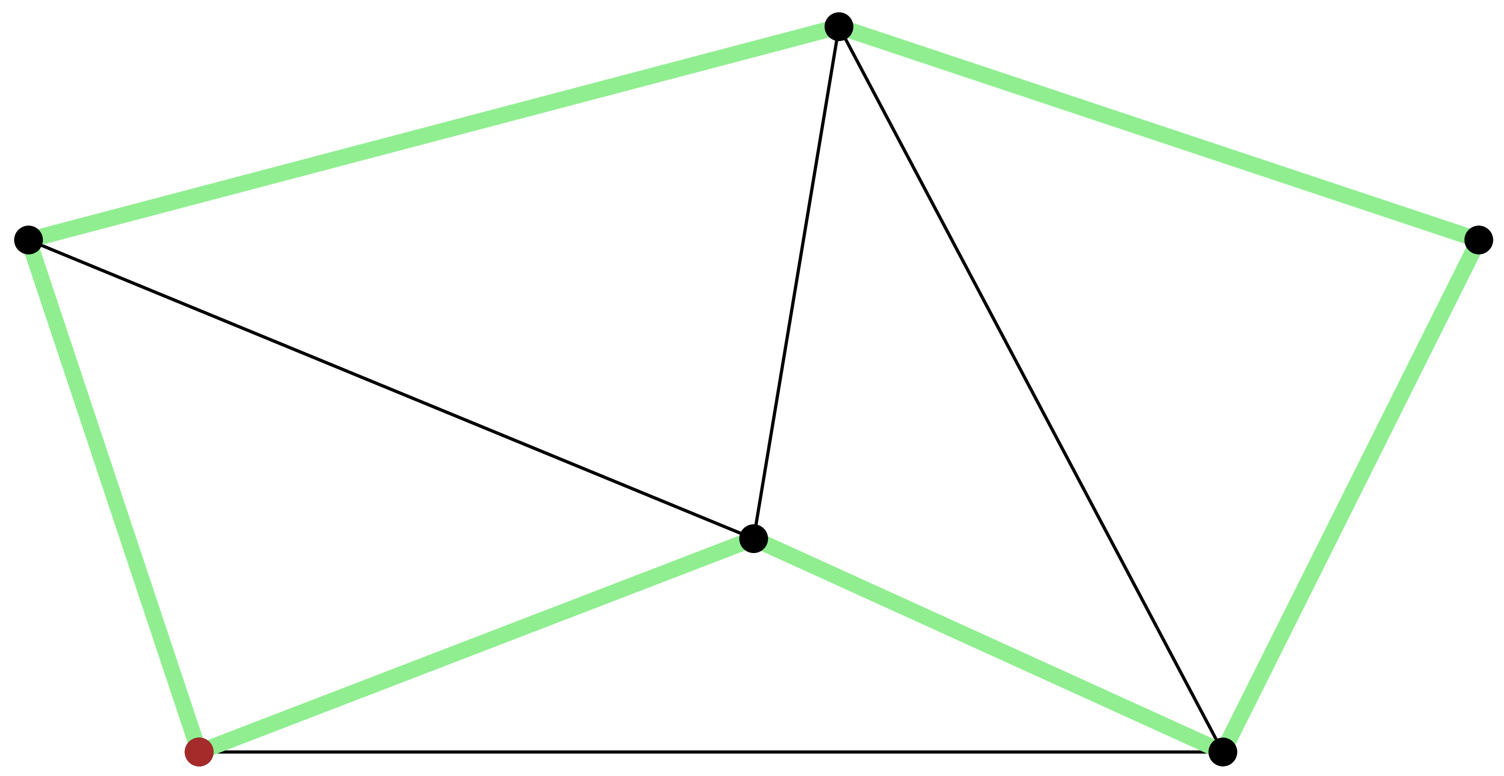

Background

The similarity between these two problems rests on a purely external analogy, but the second problem is difficult and unsolved. Nothing in general is known concerning questions of existence of a Hamiltonian cycle. Here much deeper properties of graphs must be involved than in the question of an Euler trail.

D. Kőnig, Theorie der endlichen und unendlichen Graphen 1936

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I decided to make a living on the above remark.

H. Fleischner

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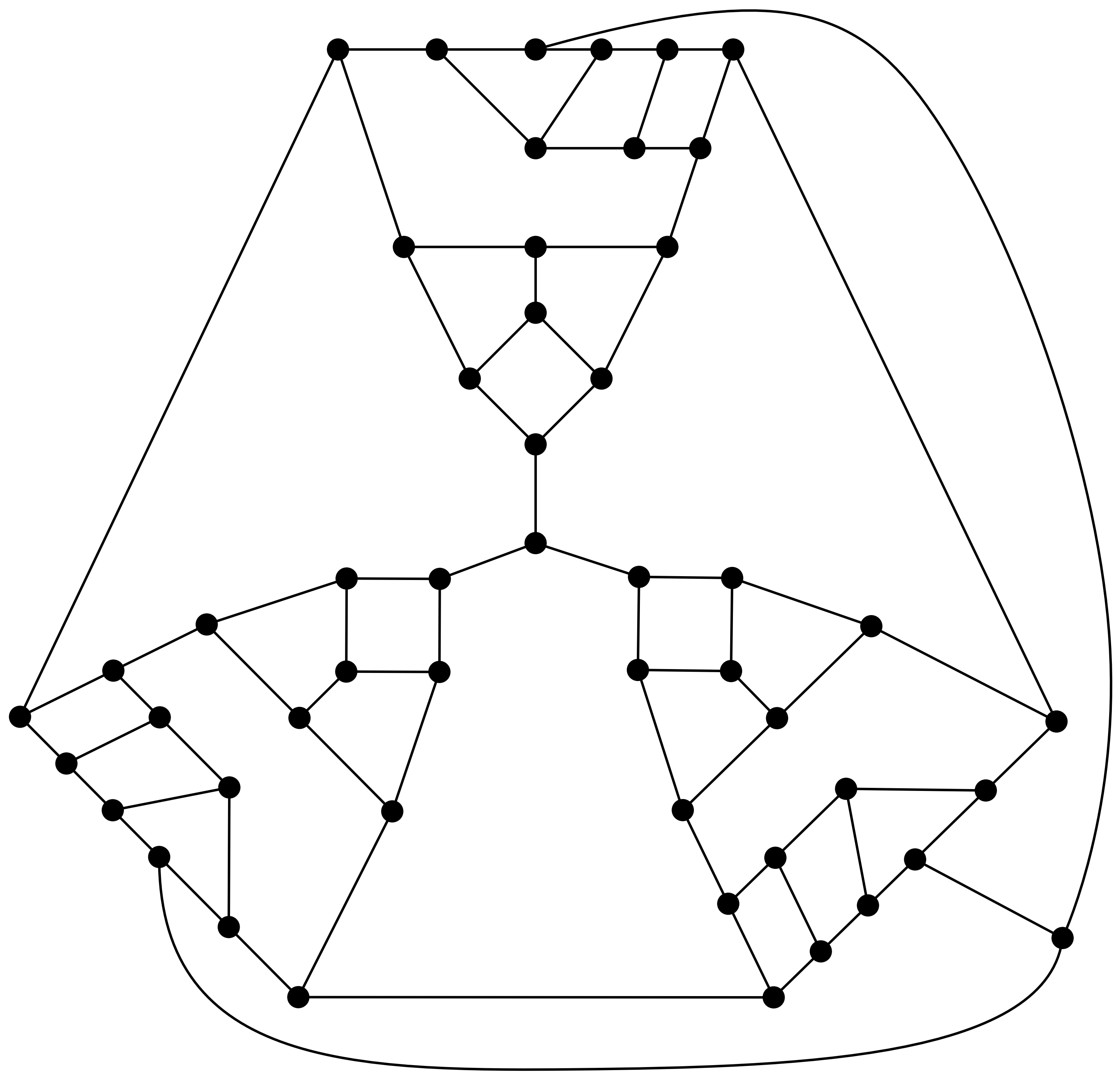

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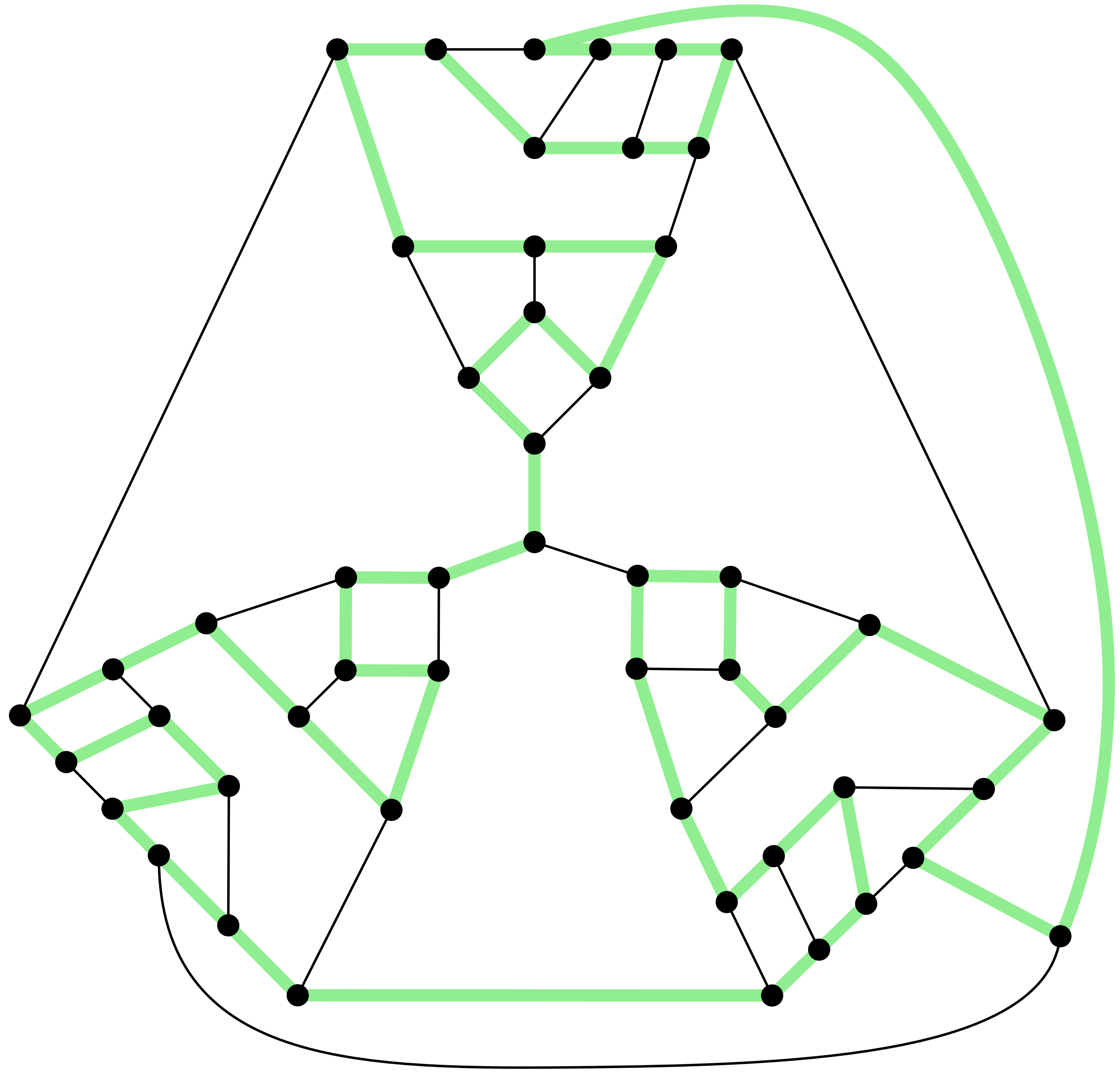

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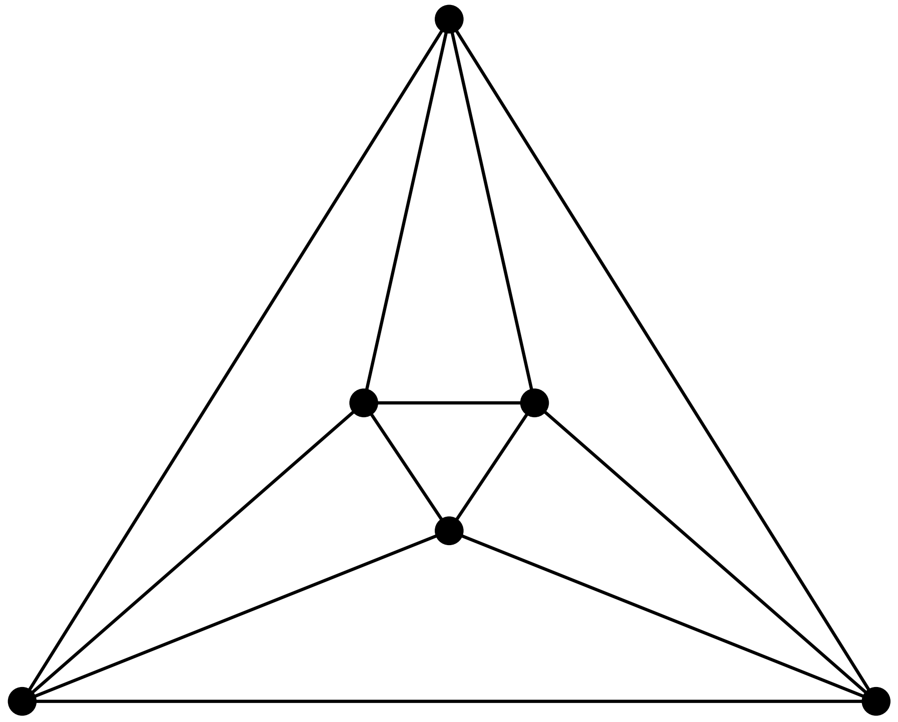

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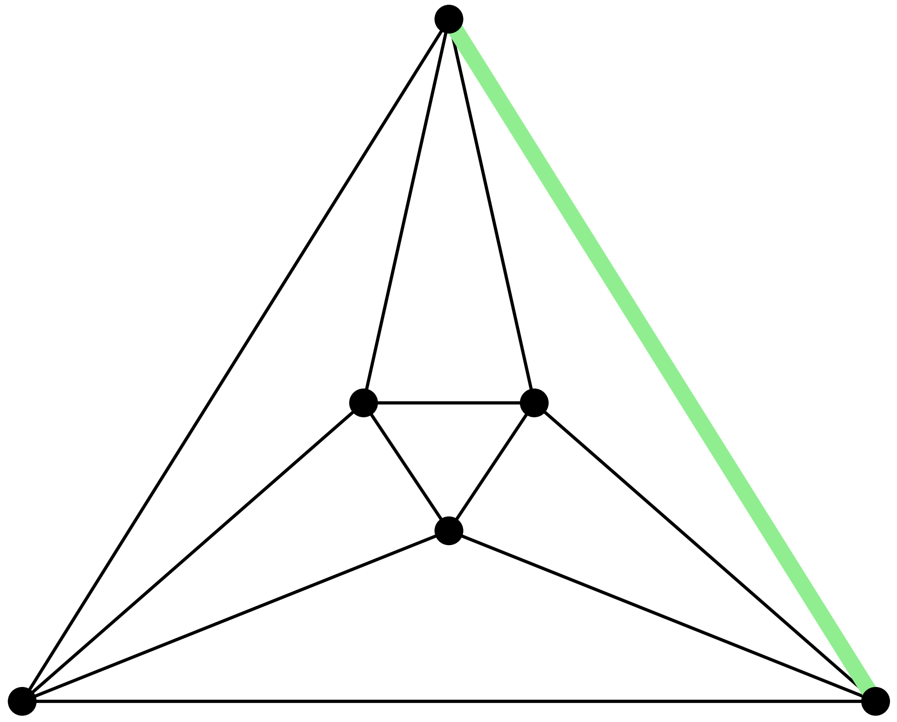

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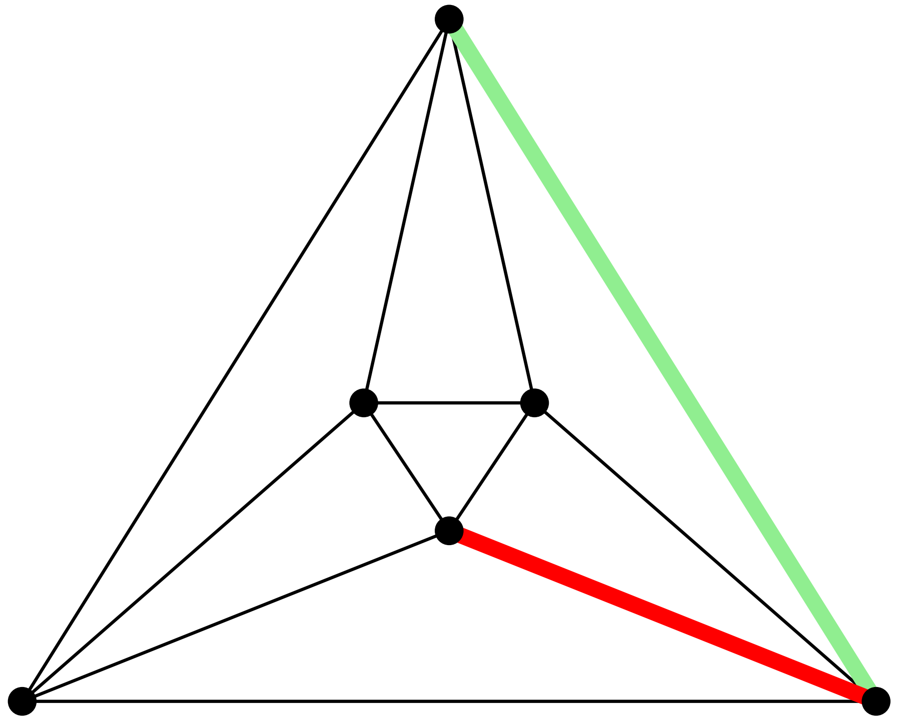

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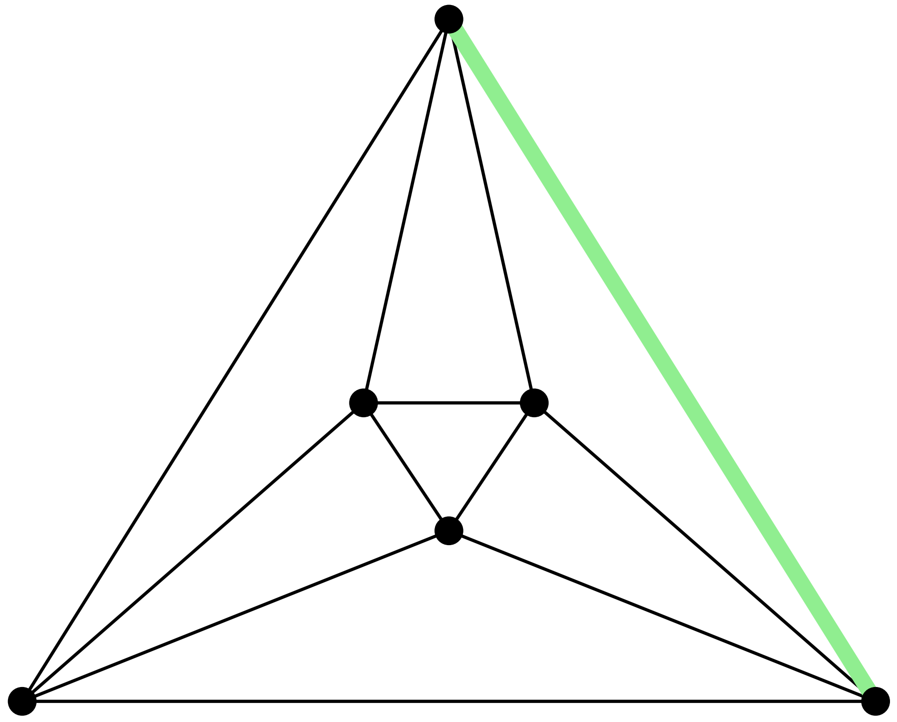

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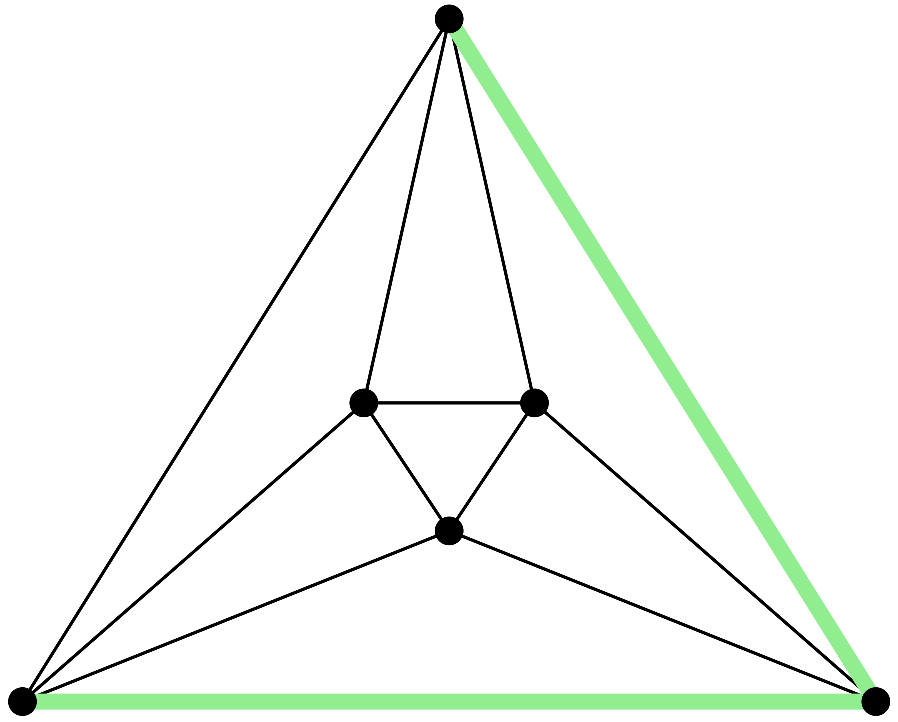

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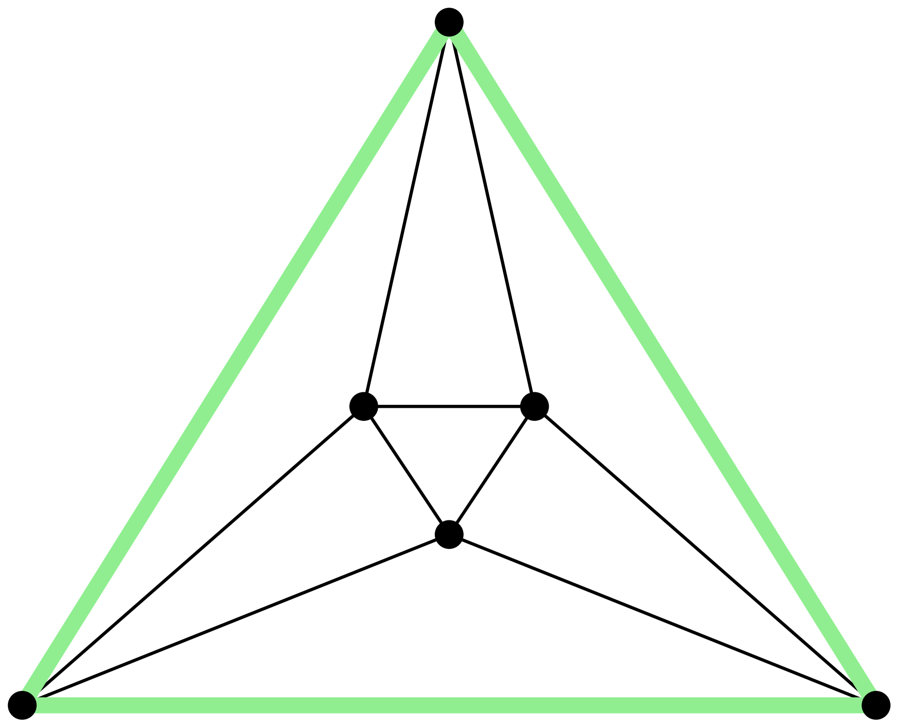

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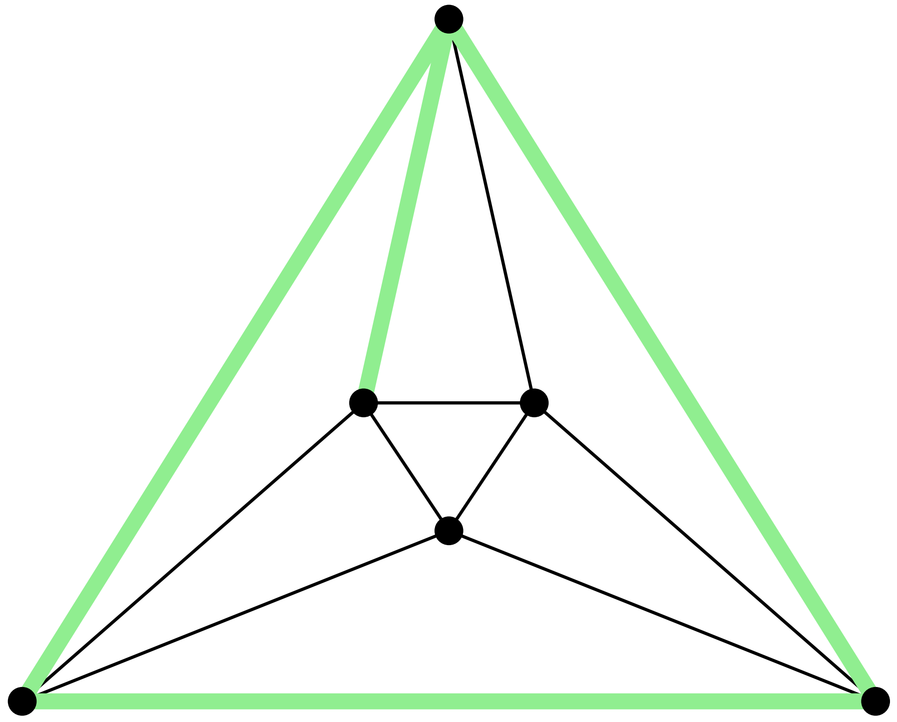

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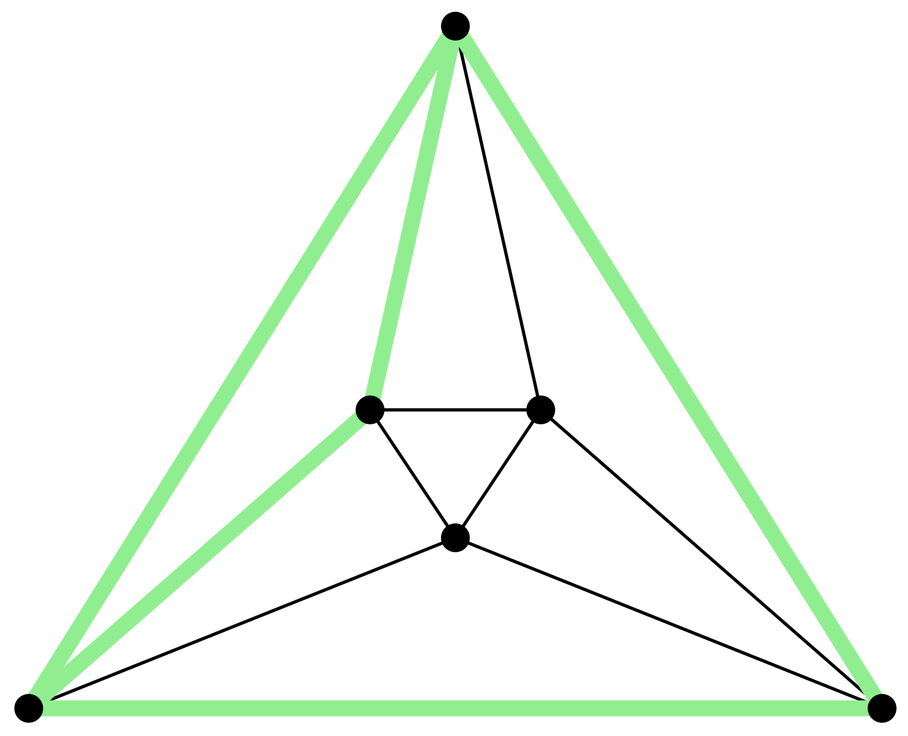

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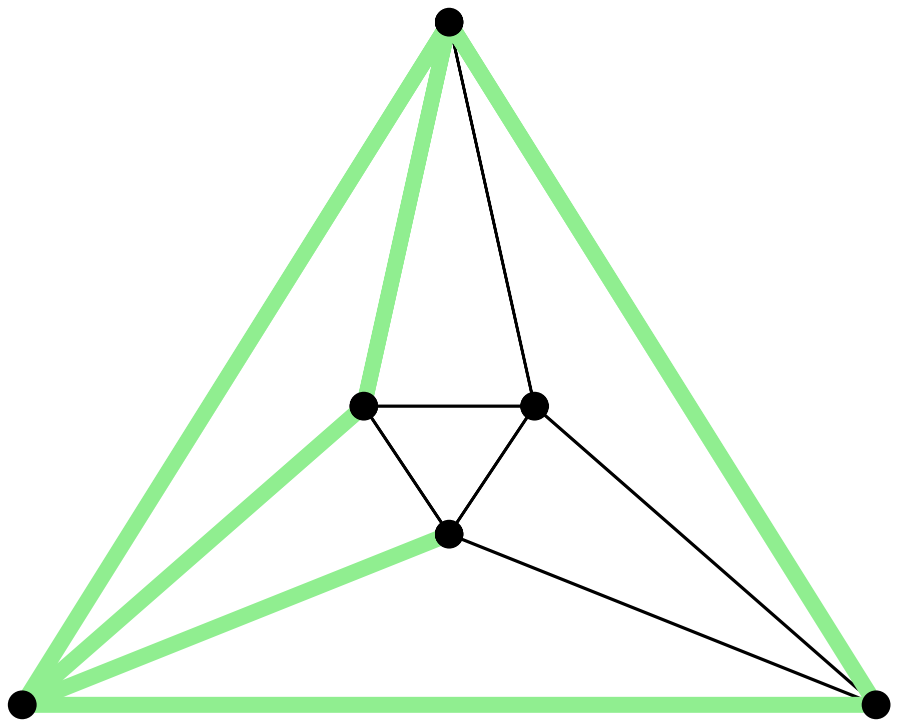

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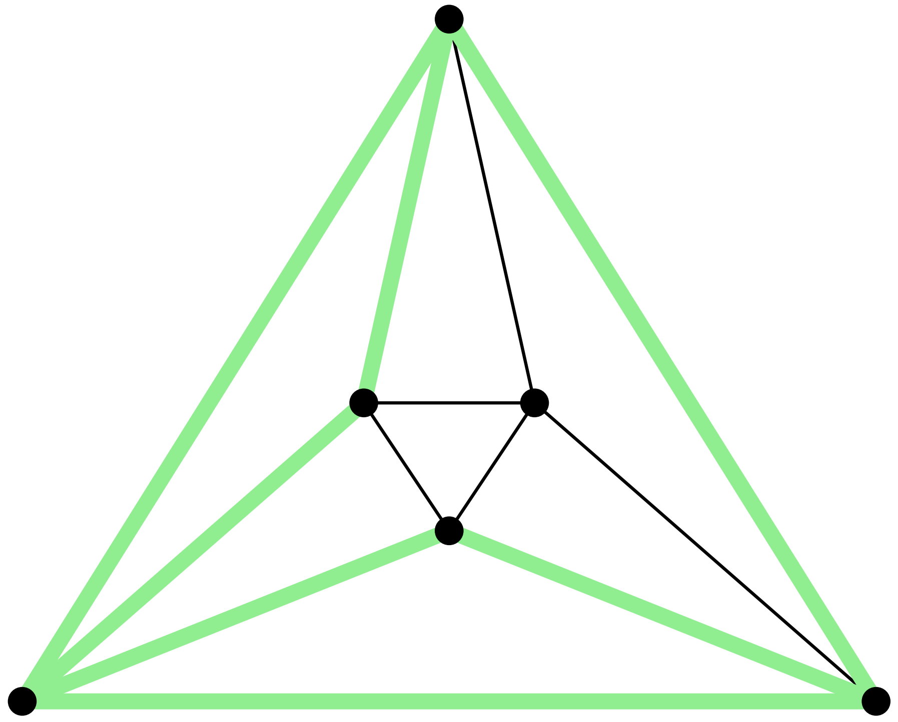

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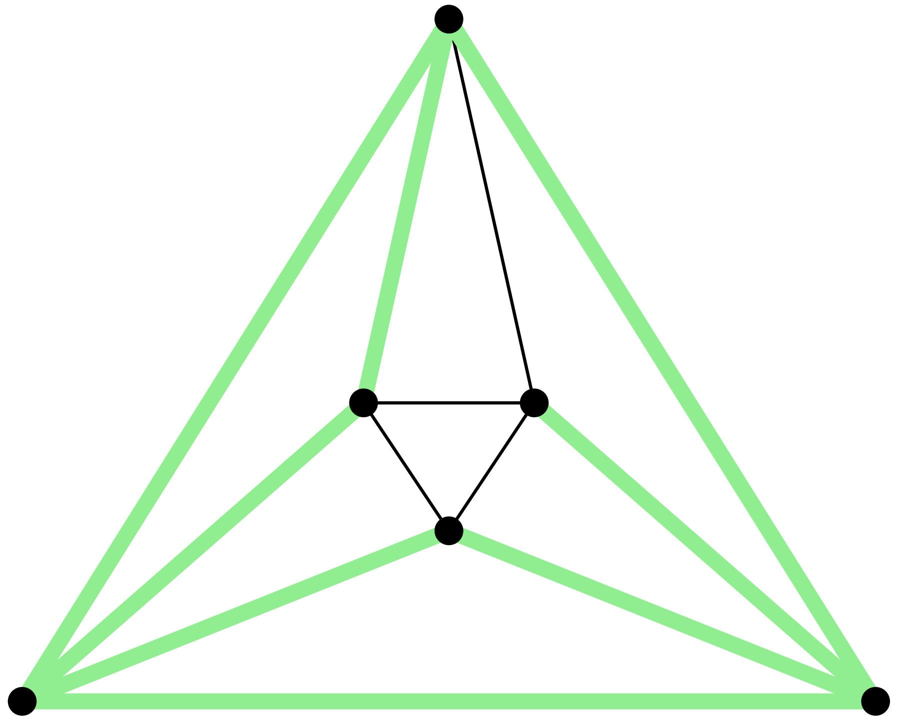

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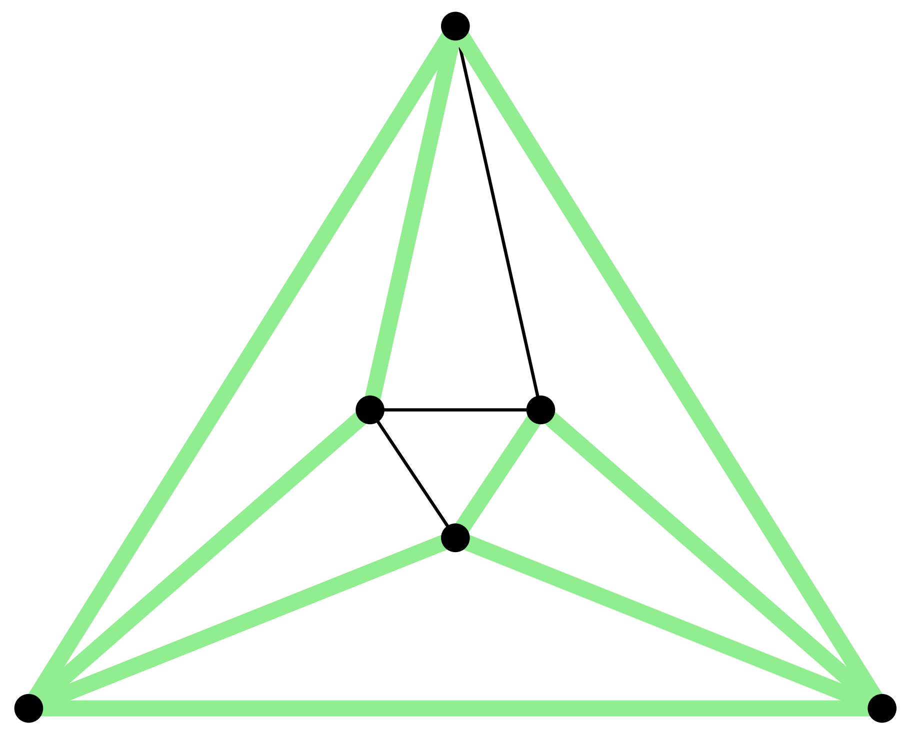

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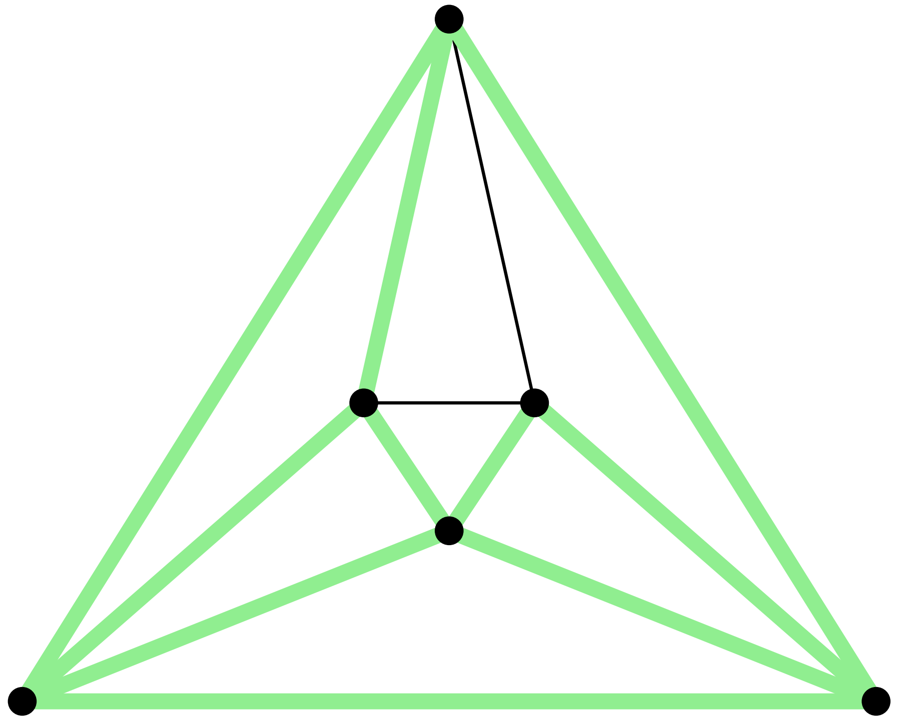

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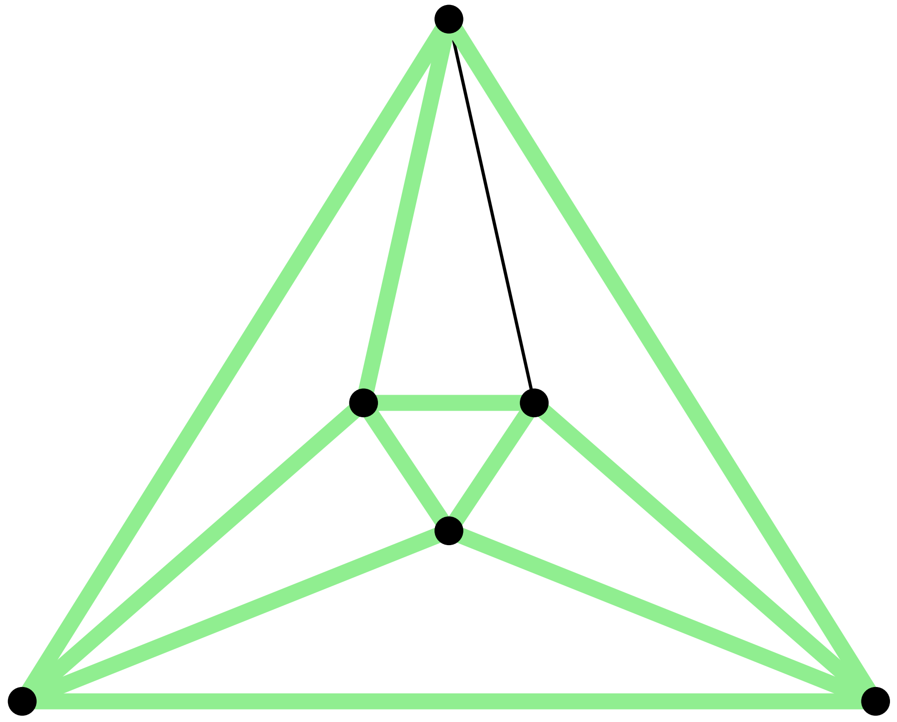

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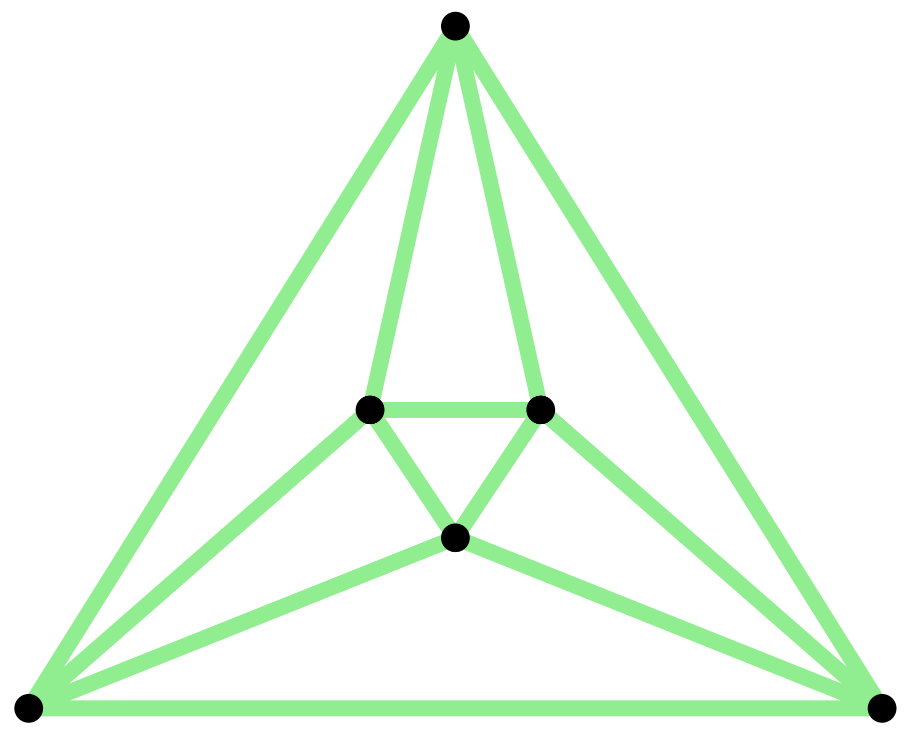

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Conjecture (Thomassen). Every 4-connected line graph is hamiltonian.

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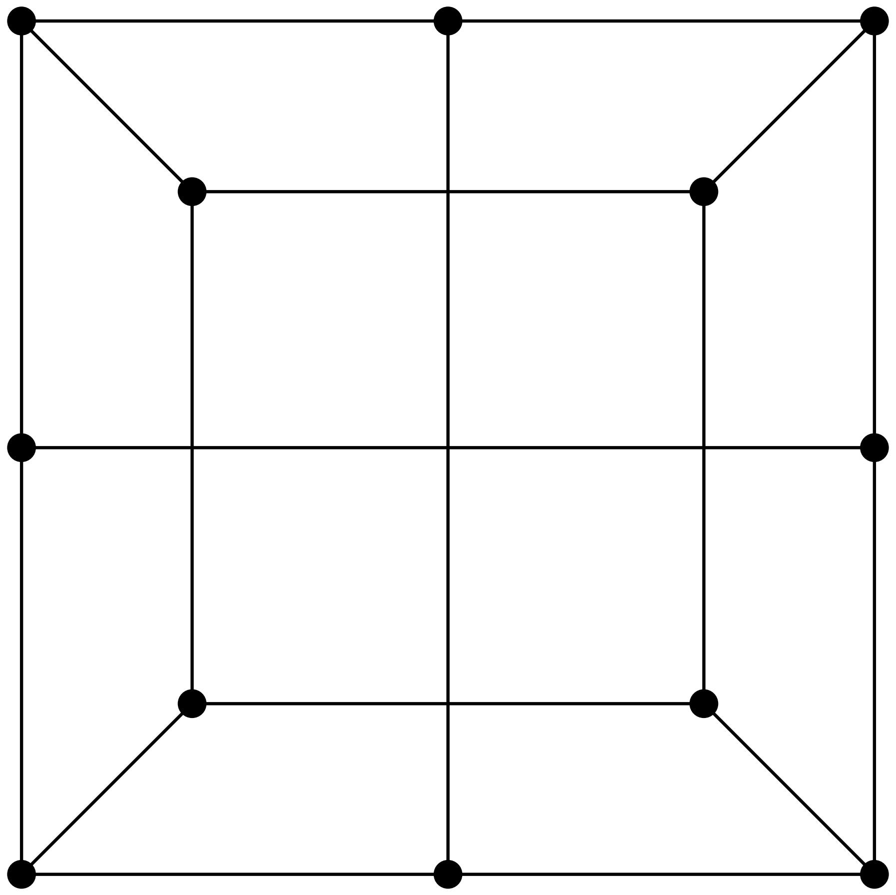

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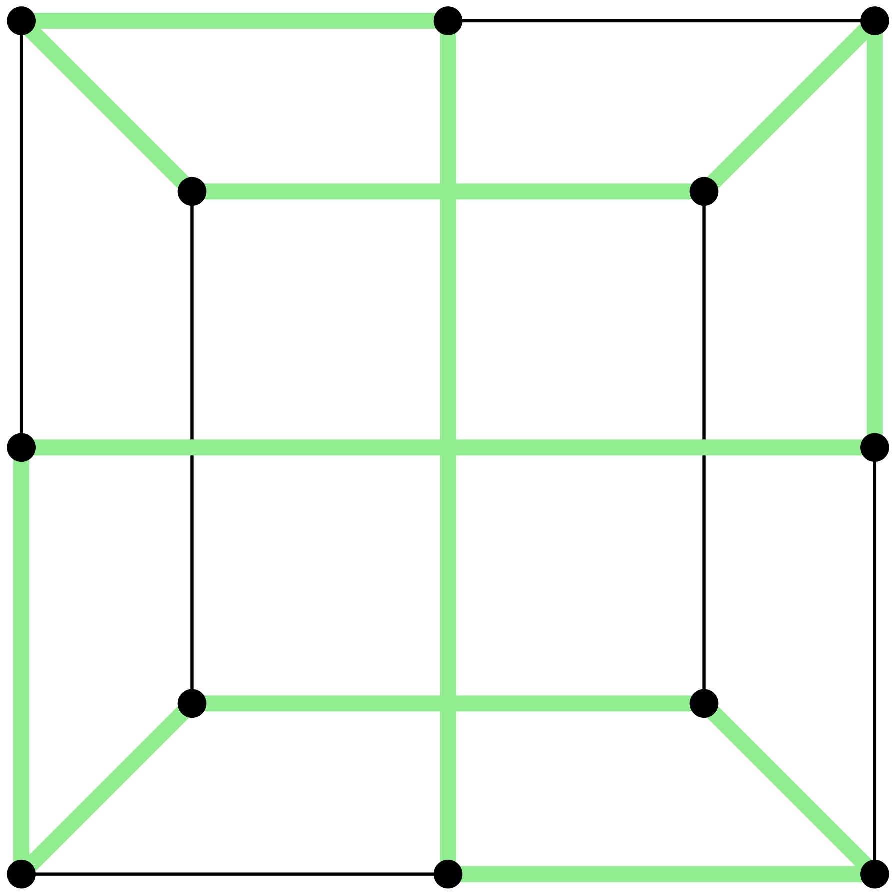

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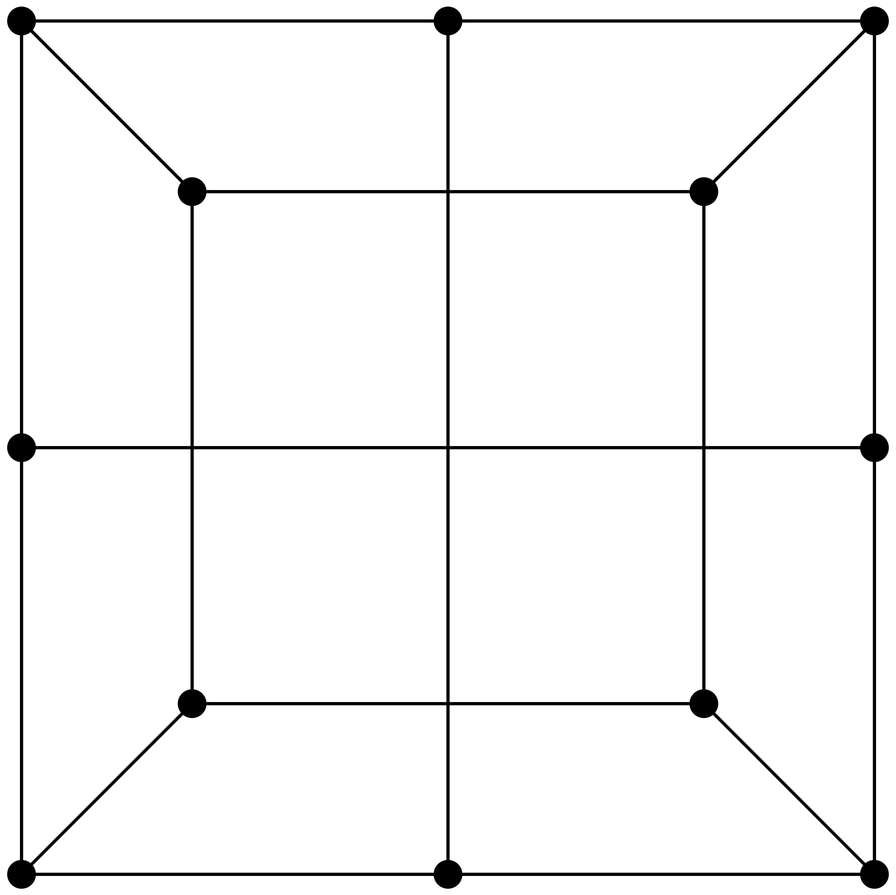

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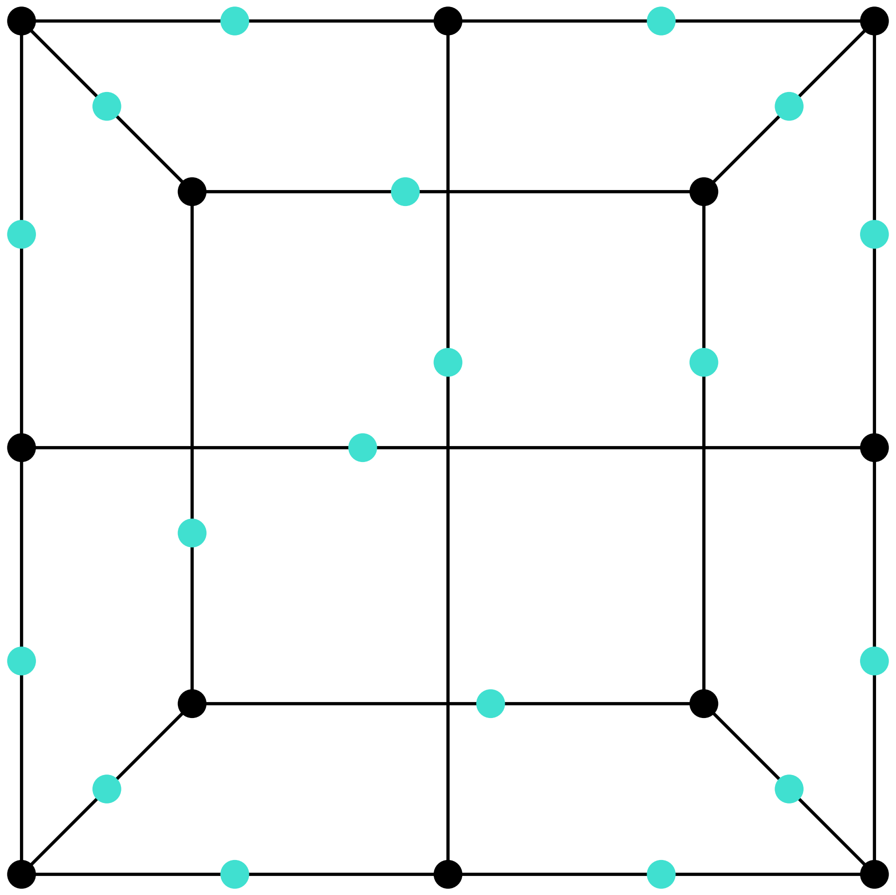

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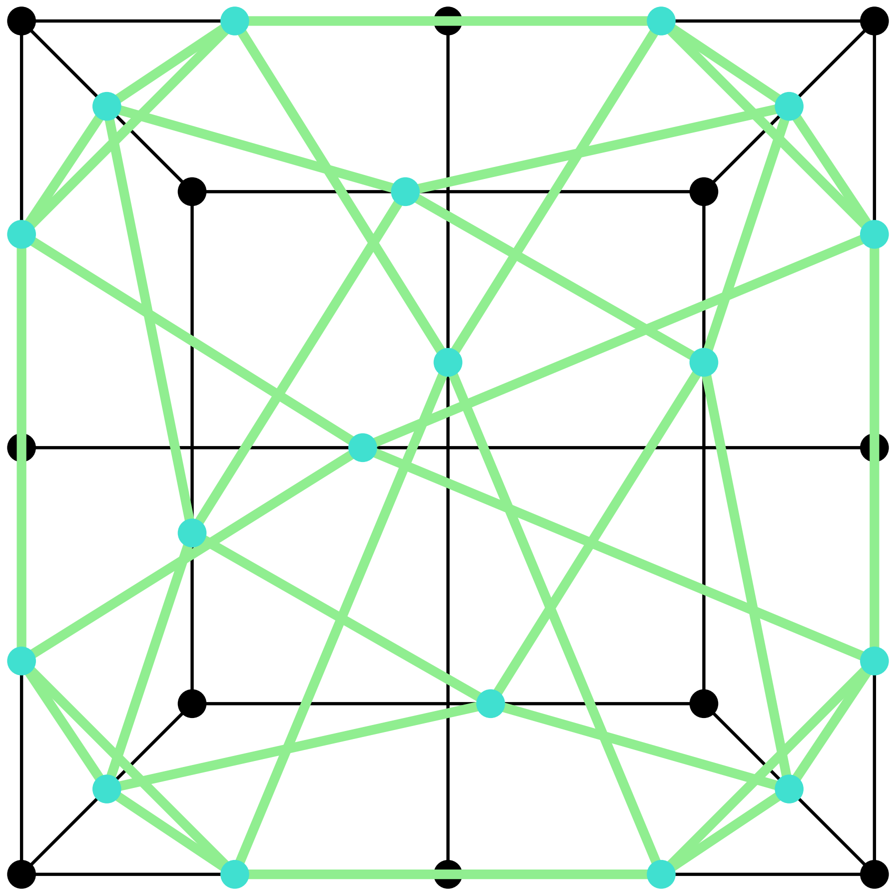

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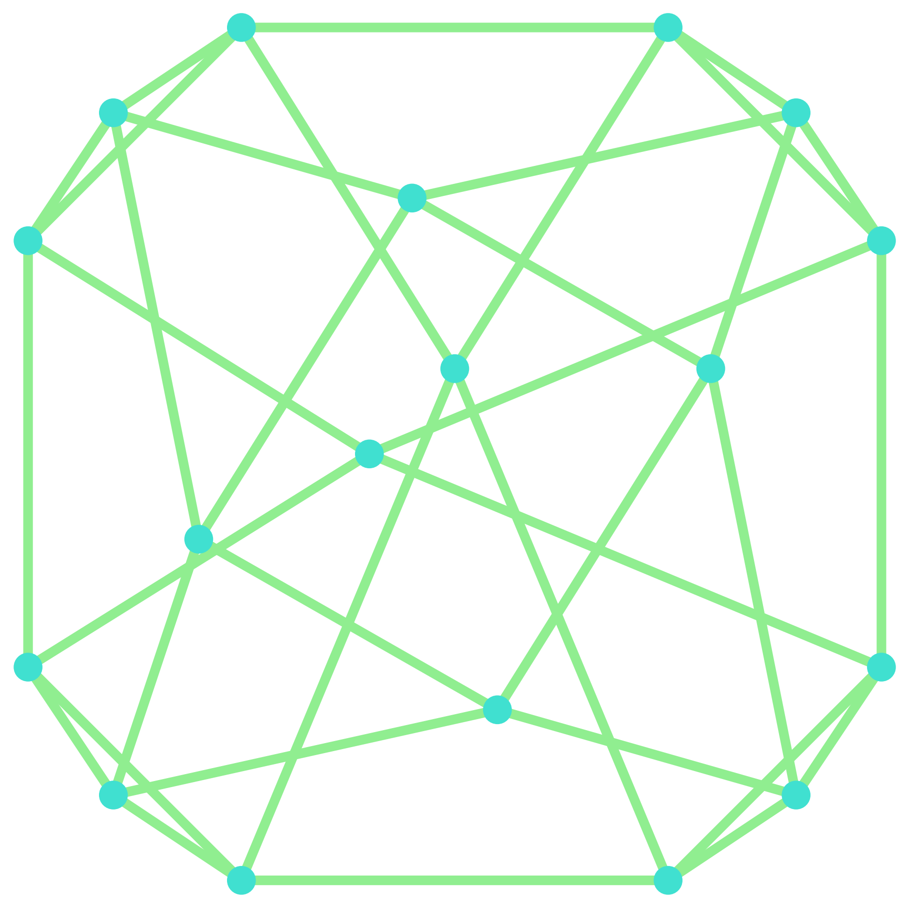

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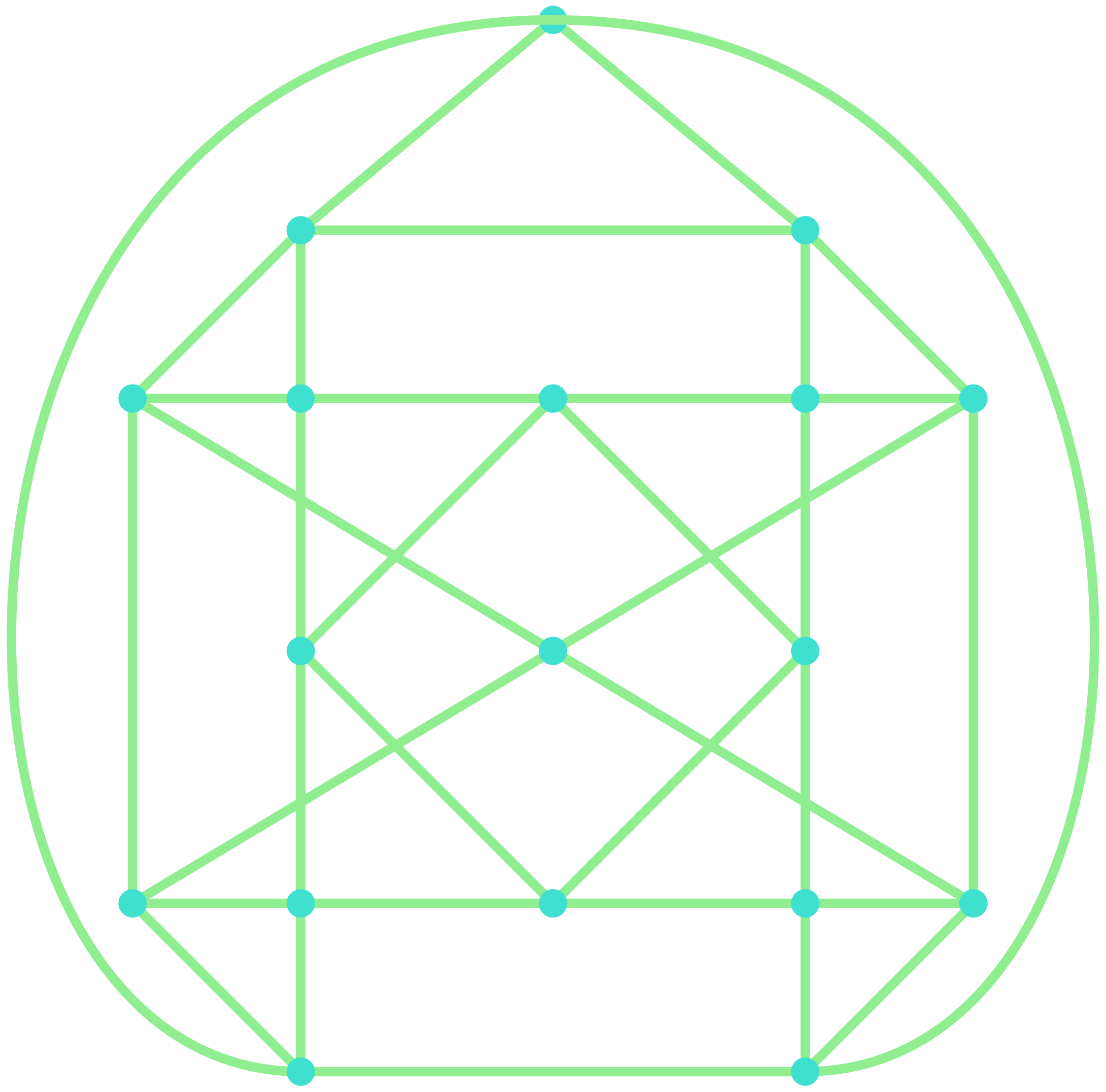

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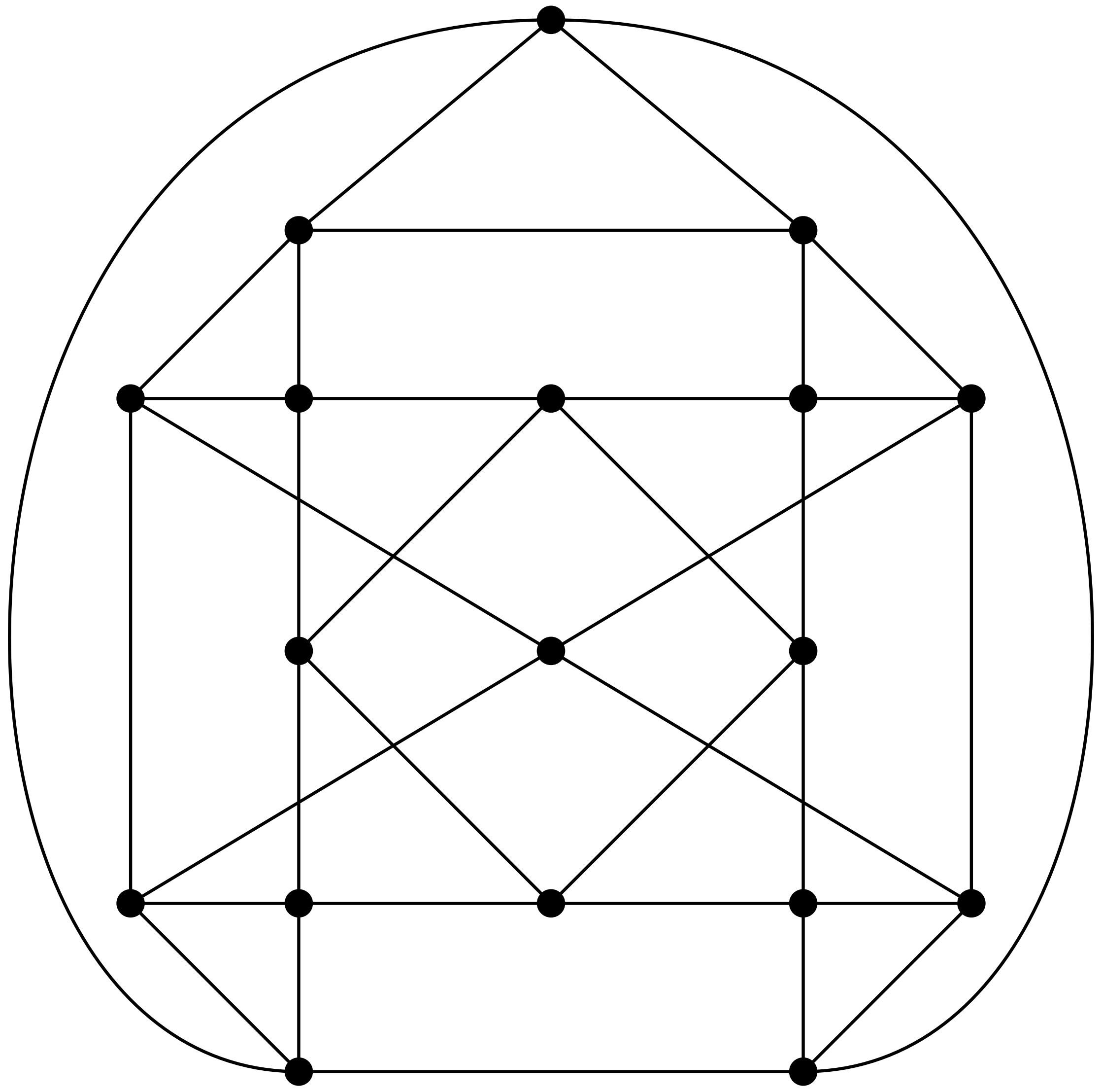

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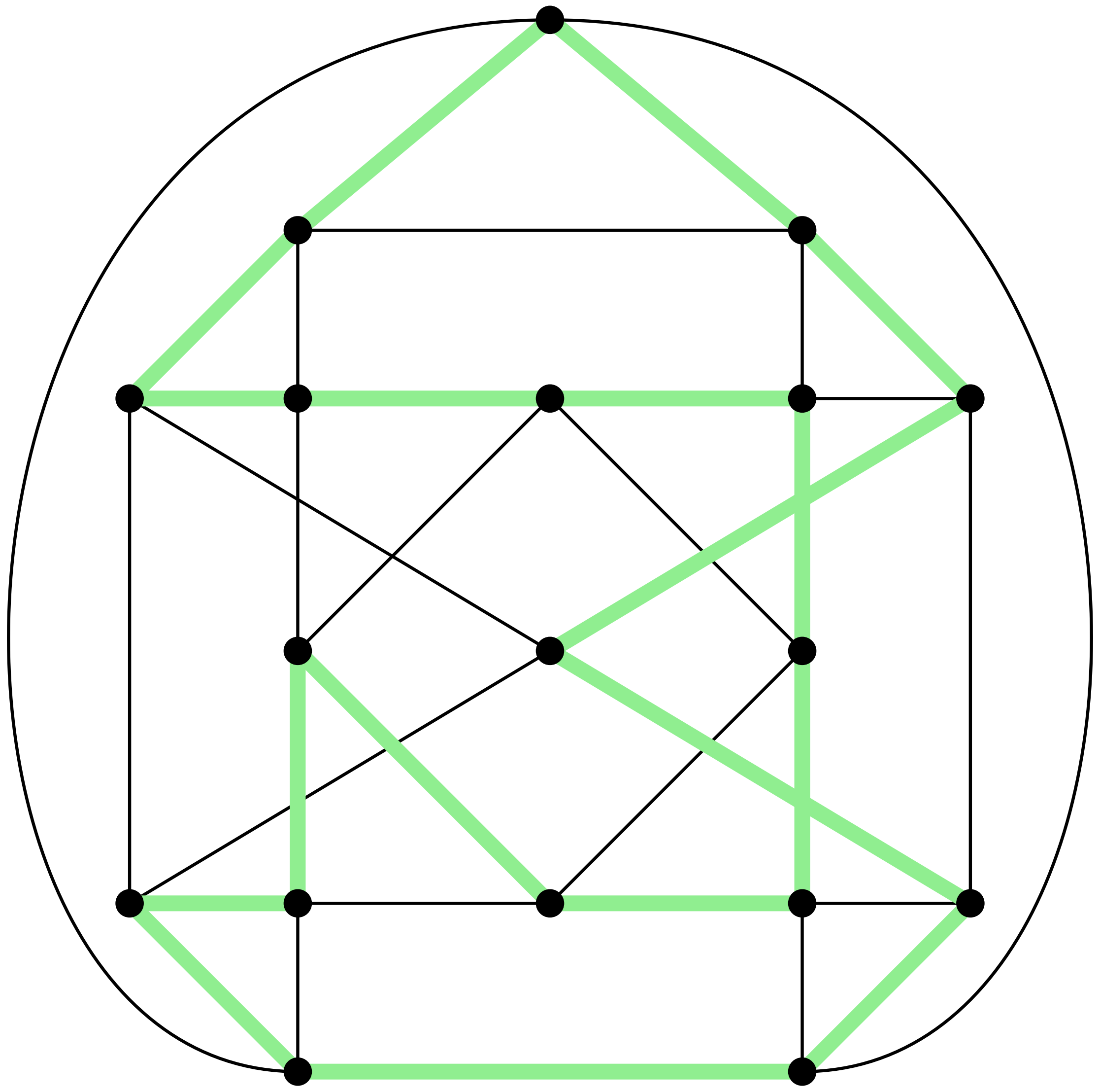

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Lemma The total graph of any given graph G is hamiltonian if and only if G has an EPS-graph

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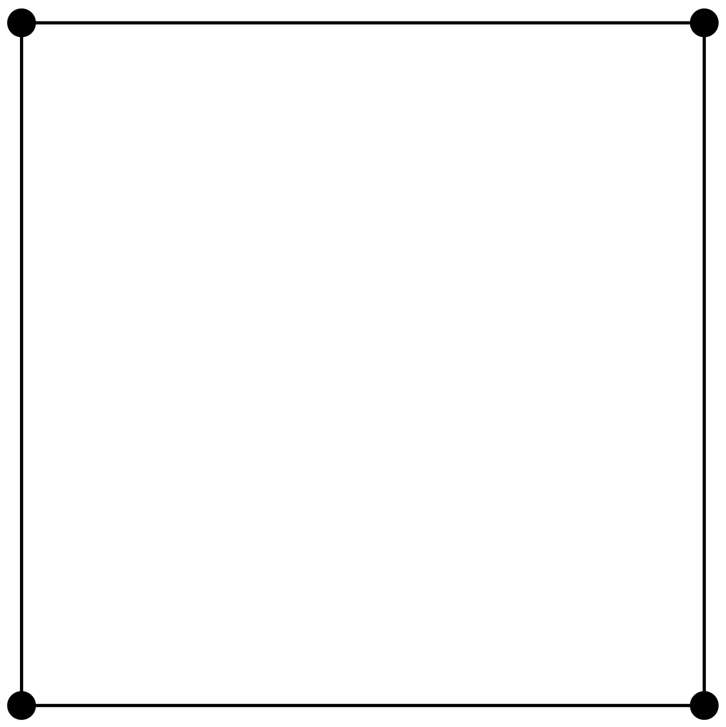

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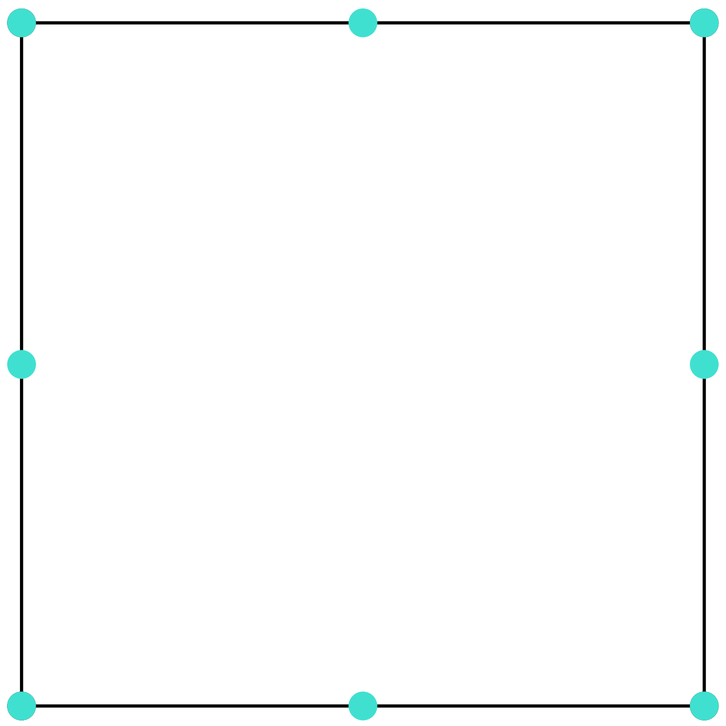

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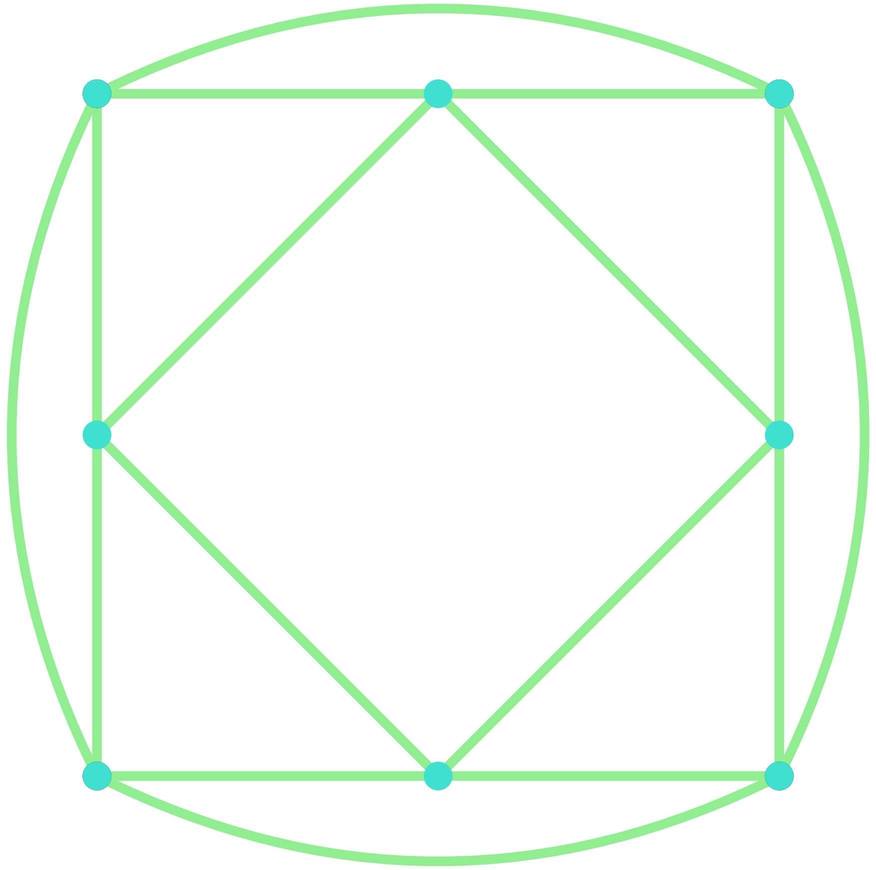

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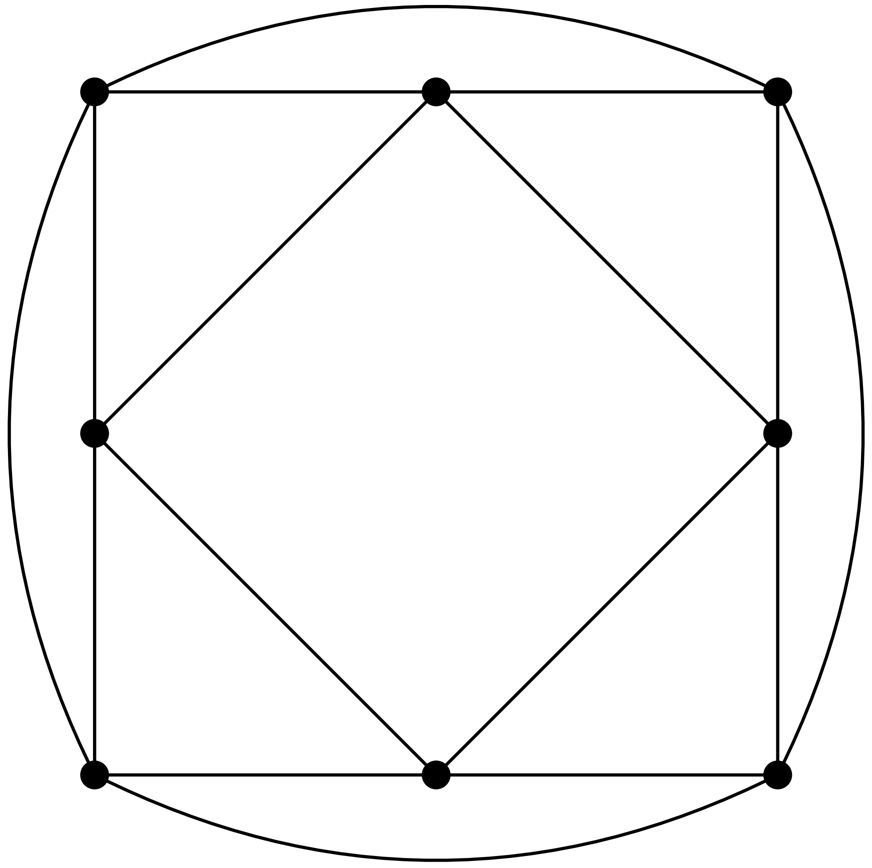

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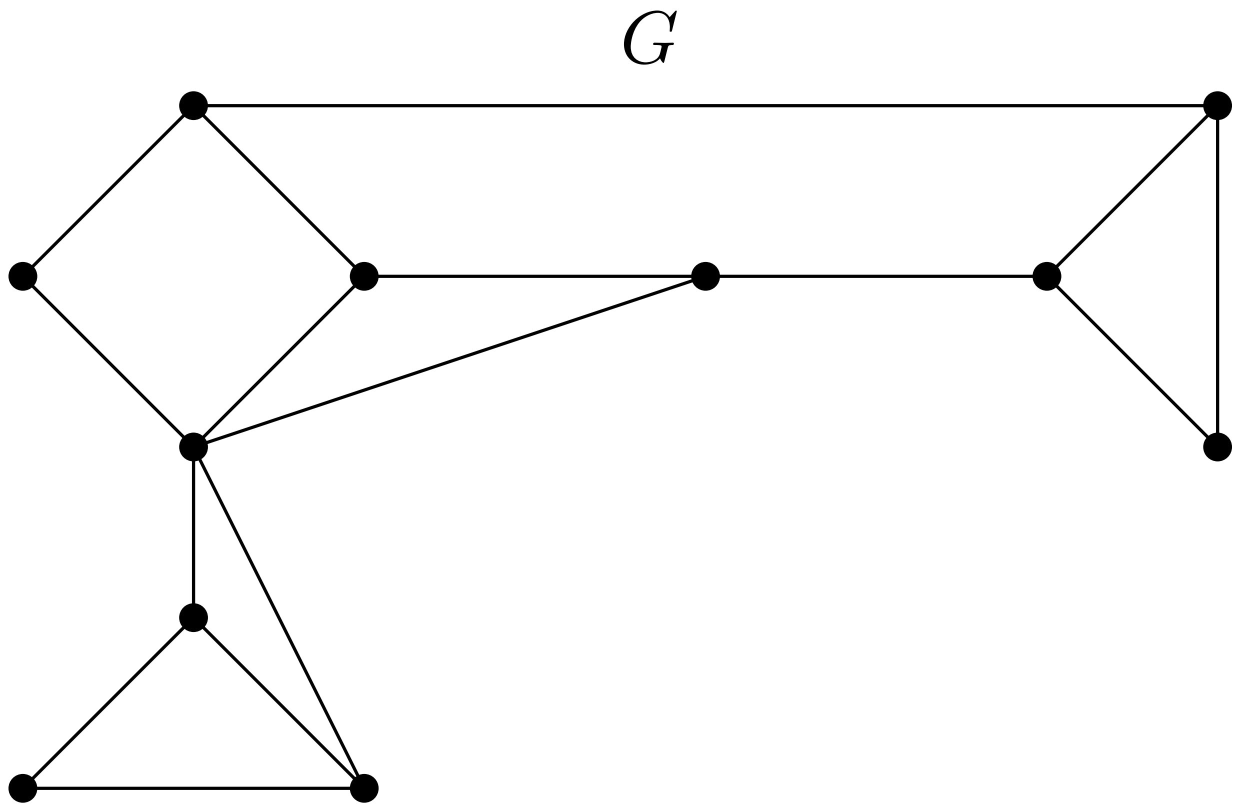

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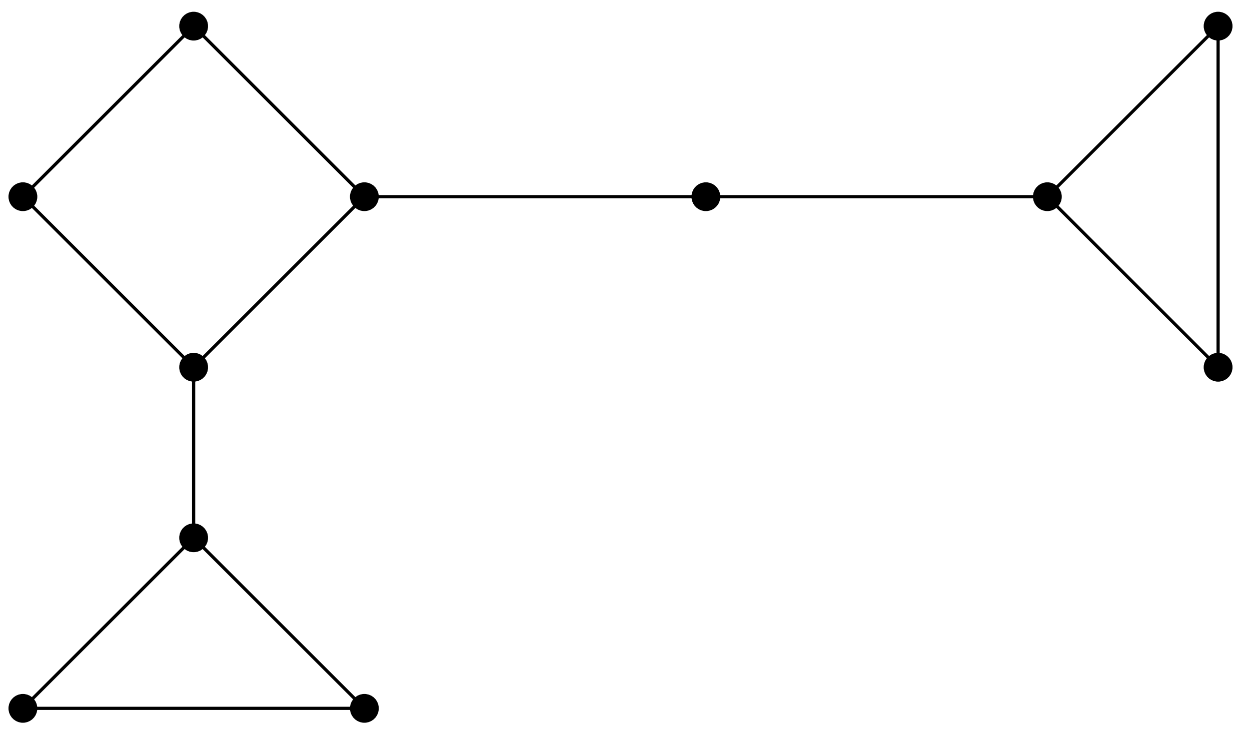

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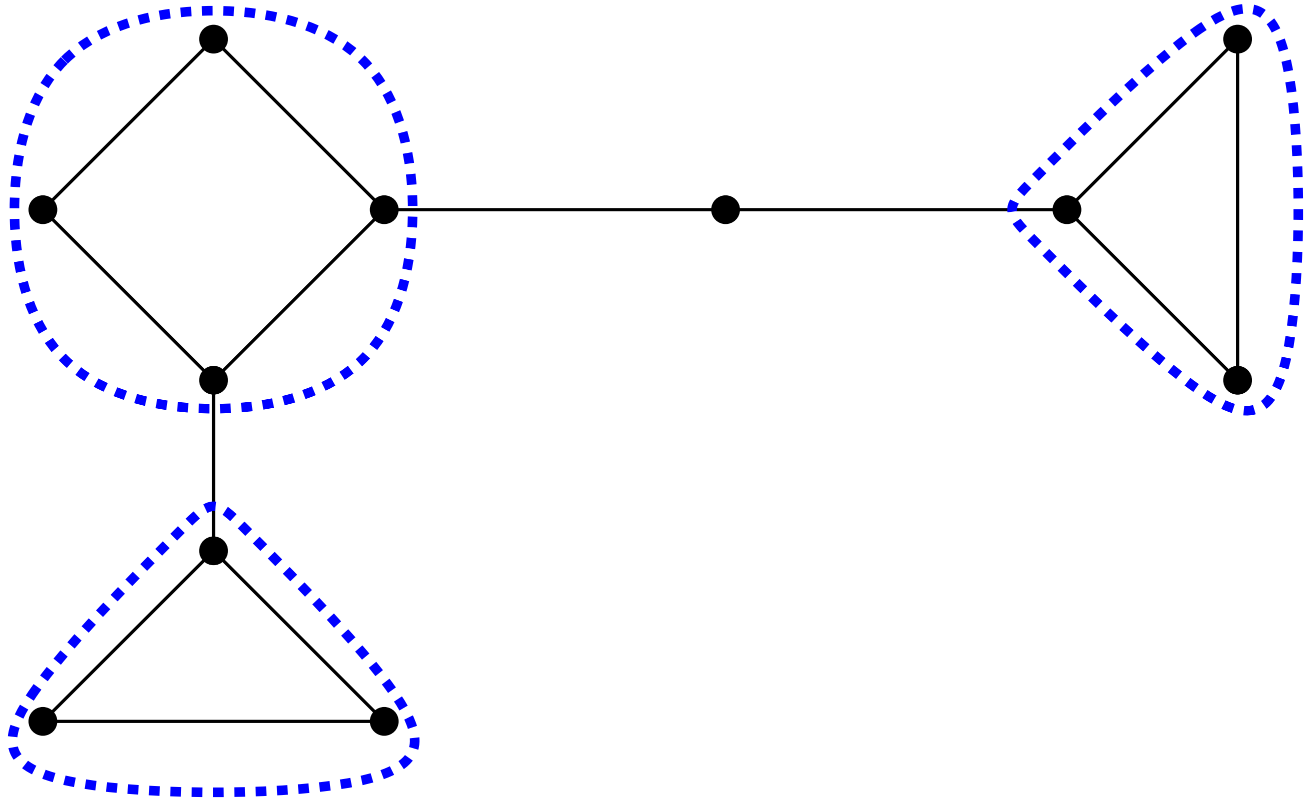

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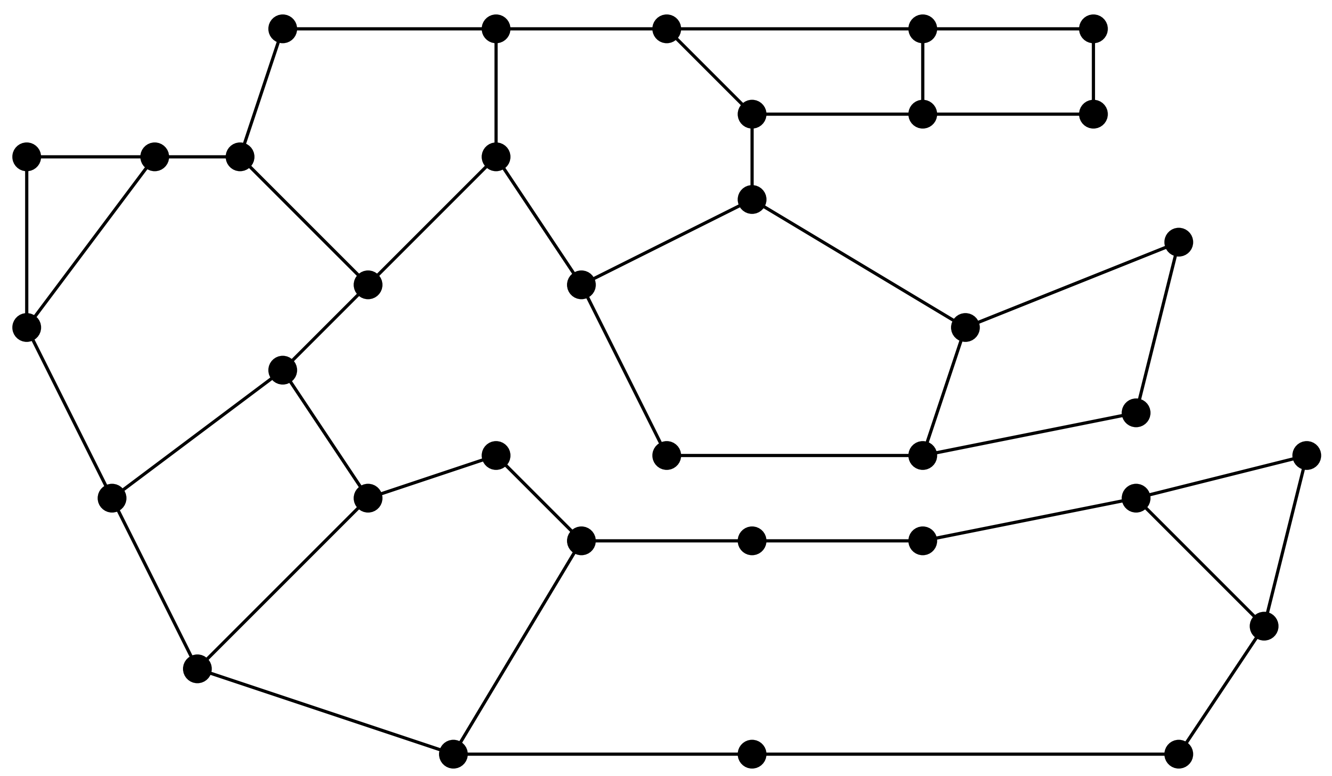

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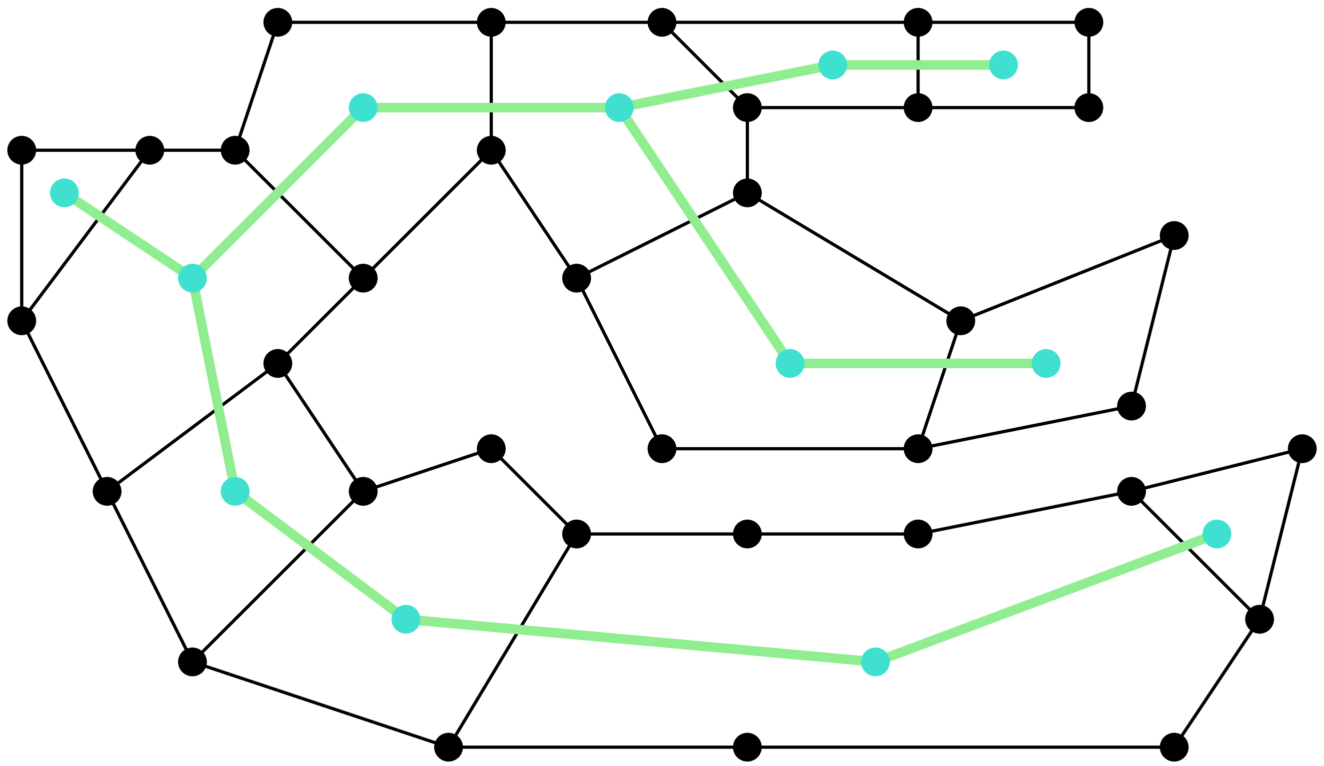

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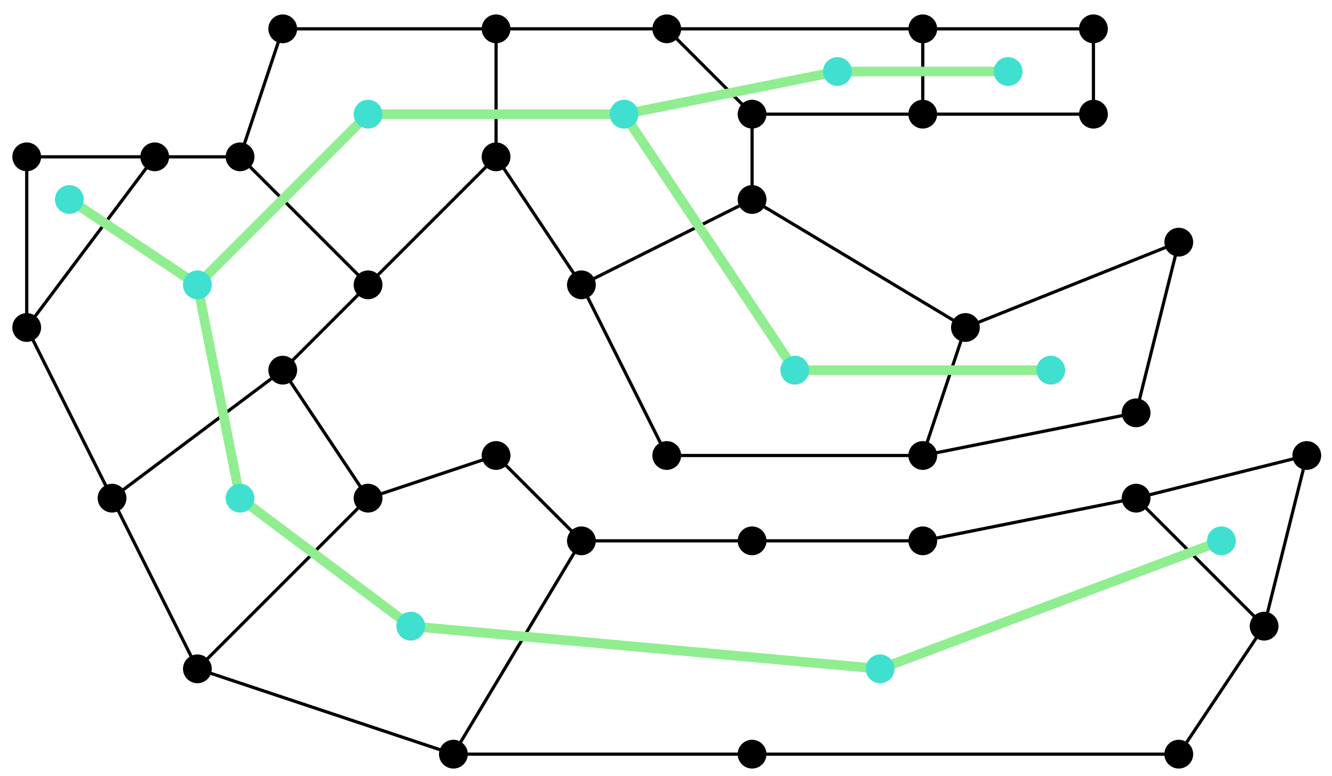

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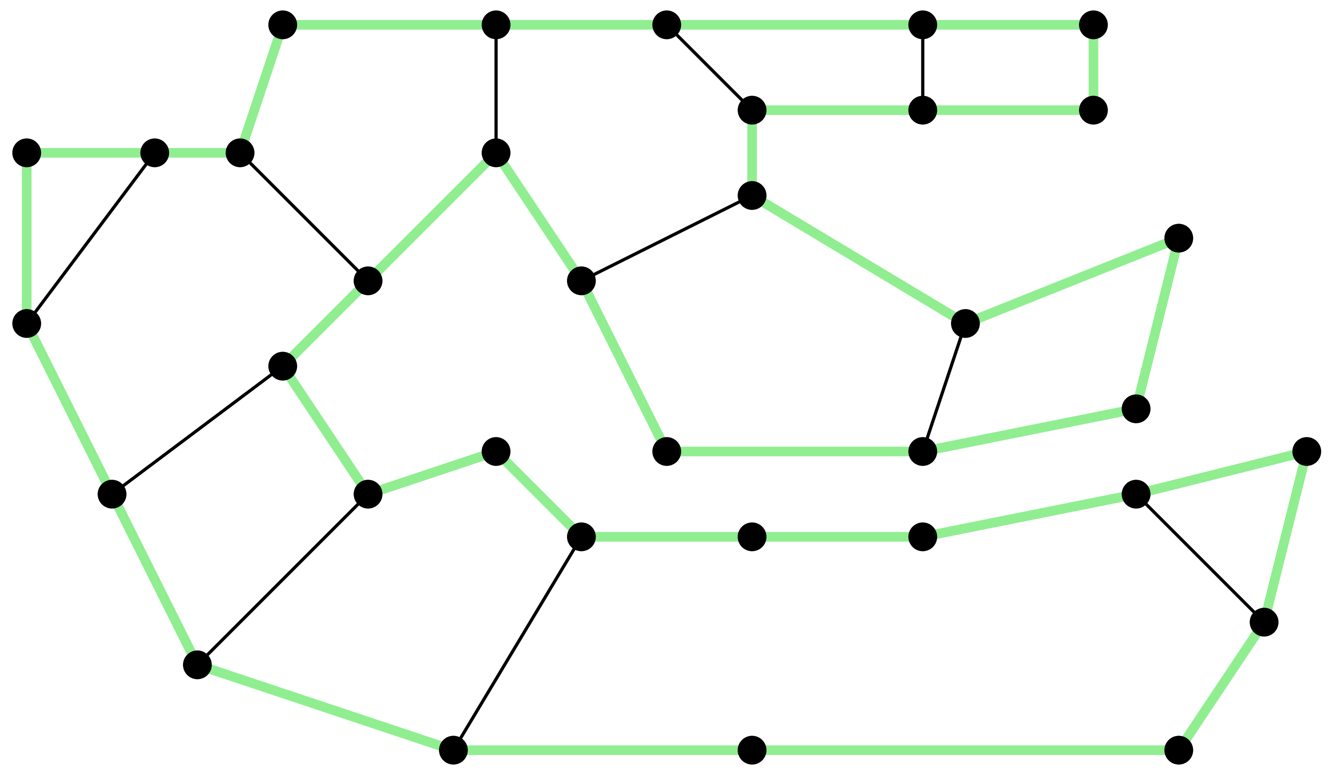

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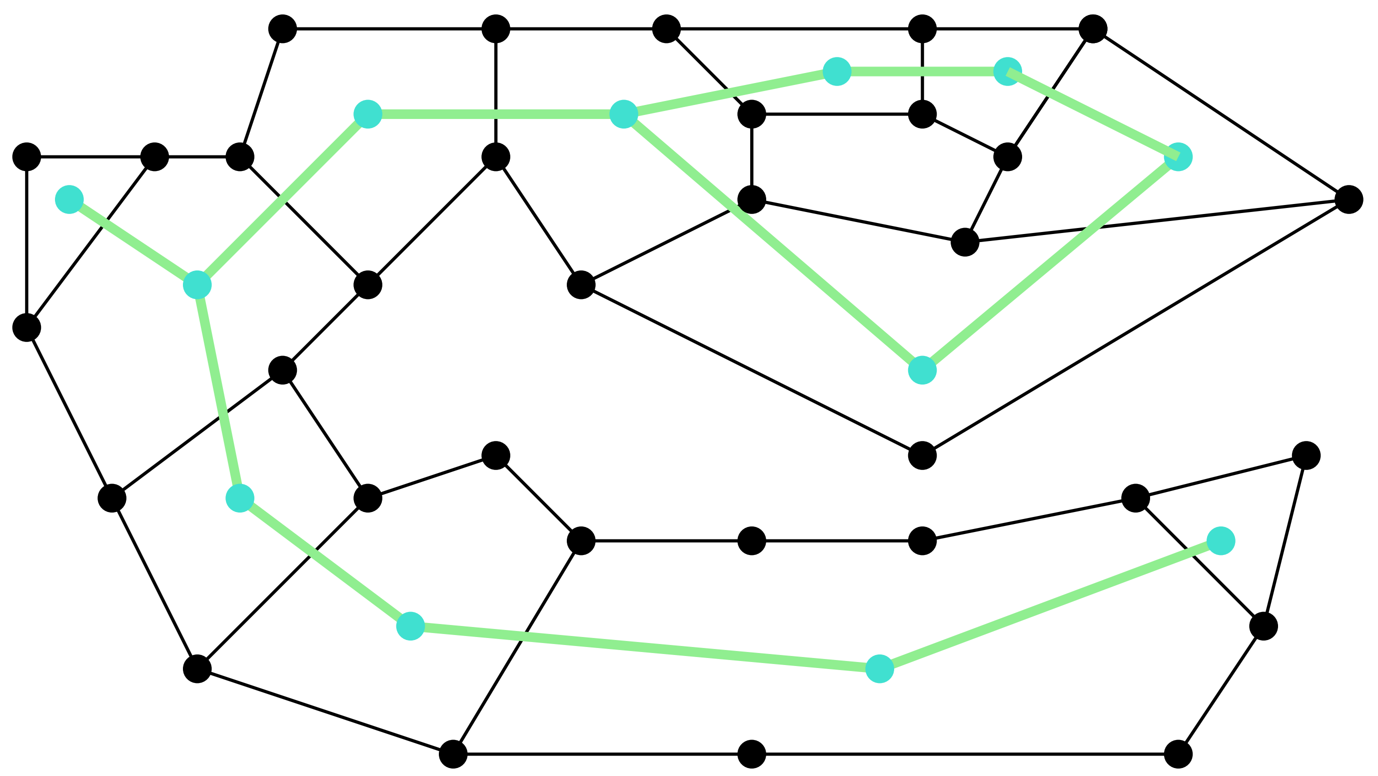

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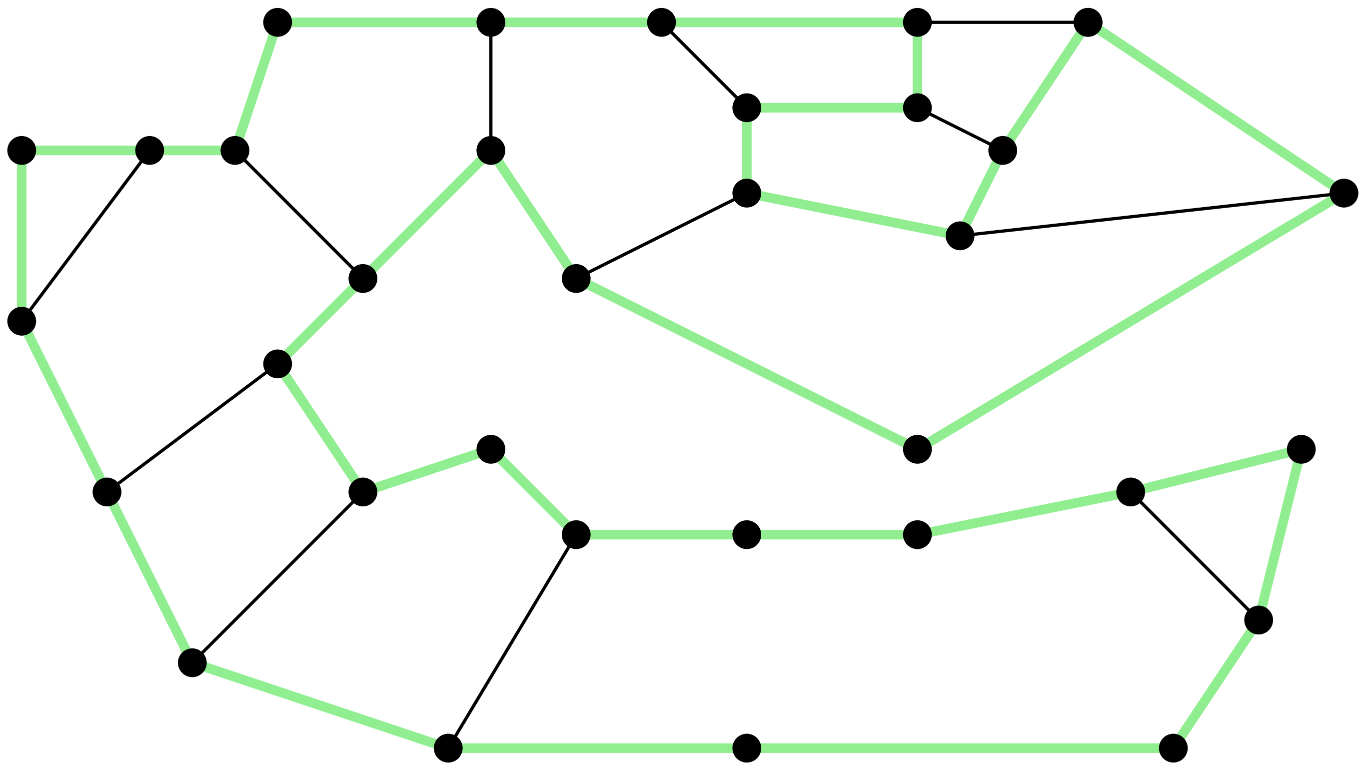

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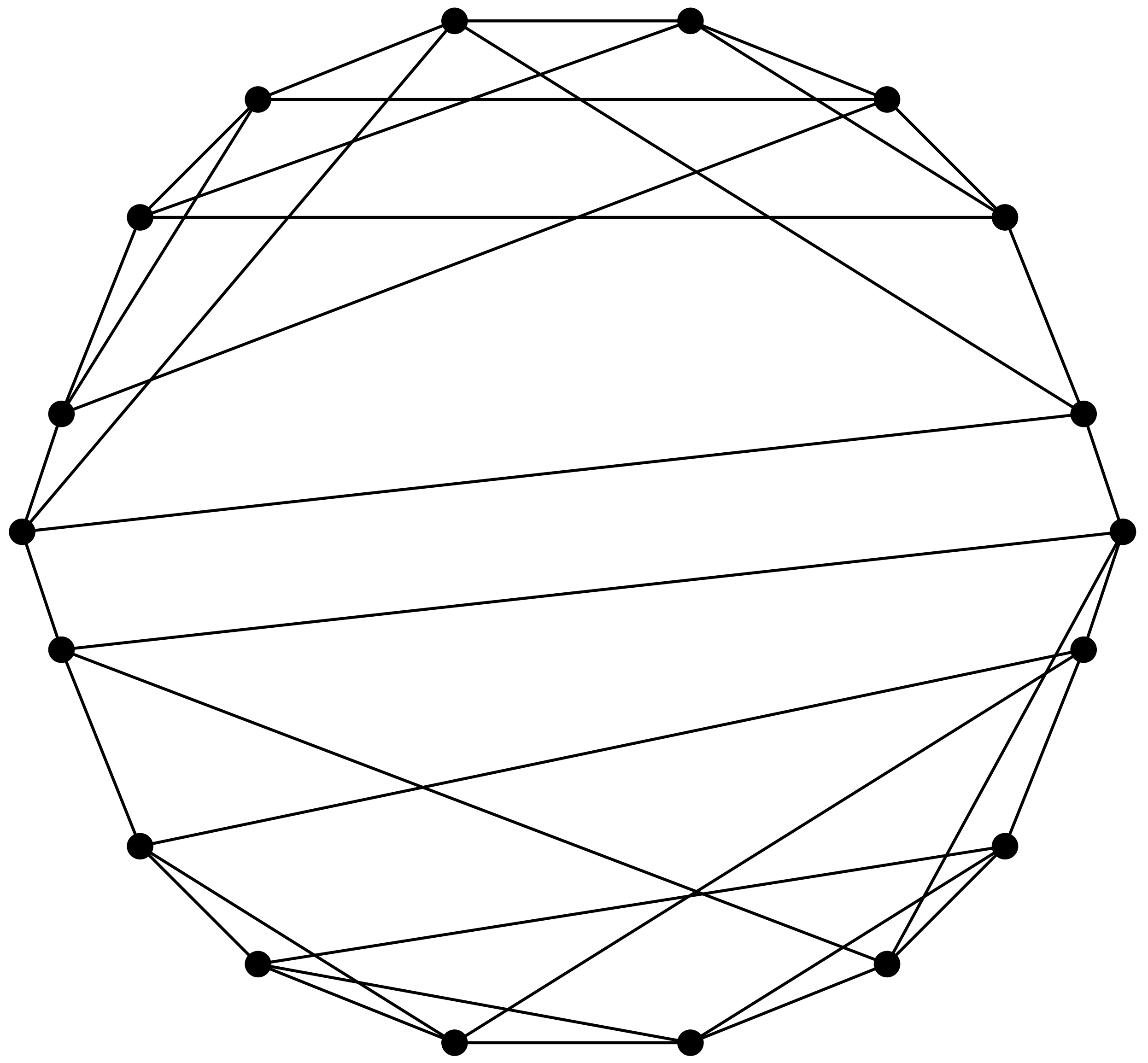

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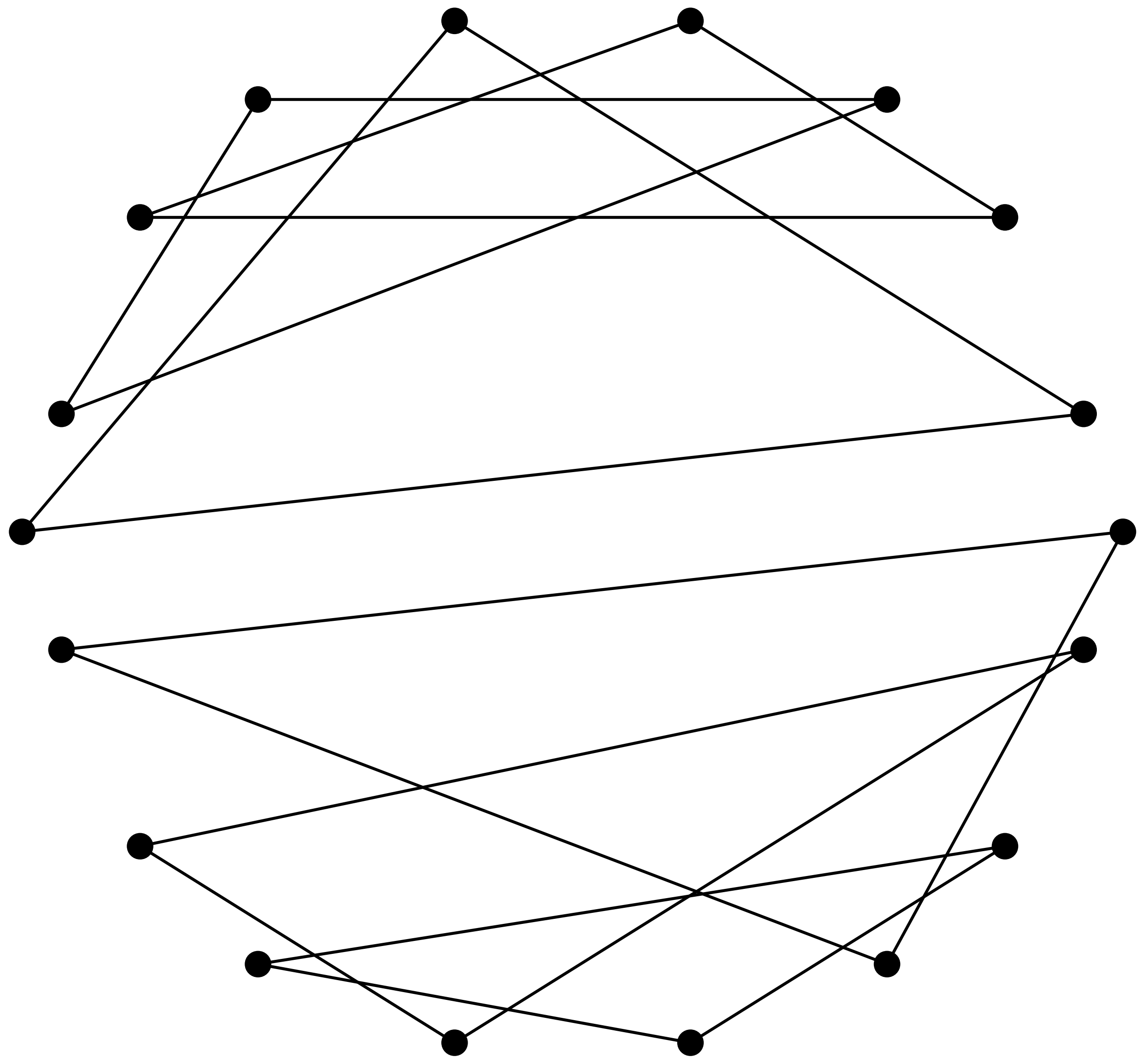

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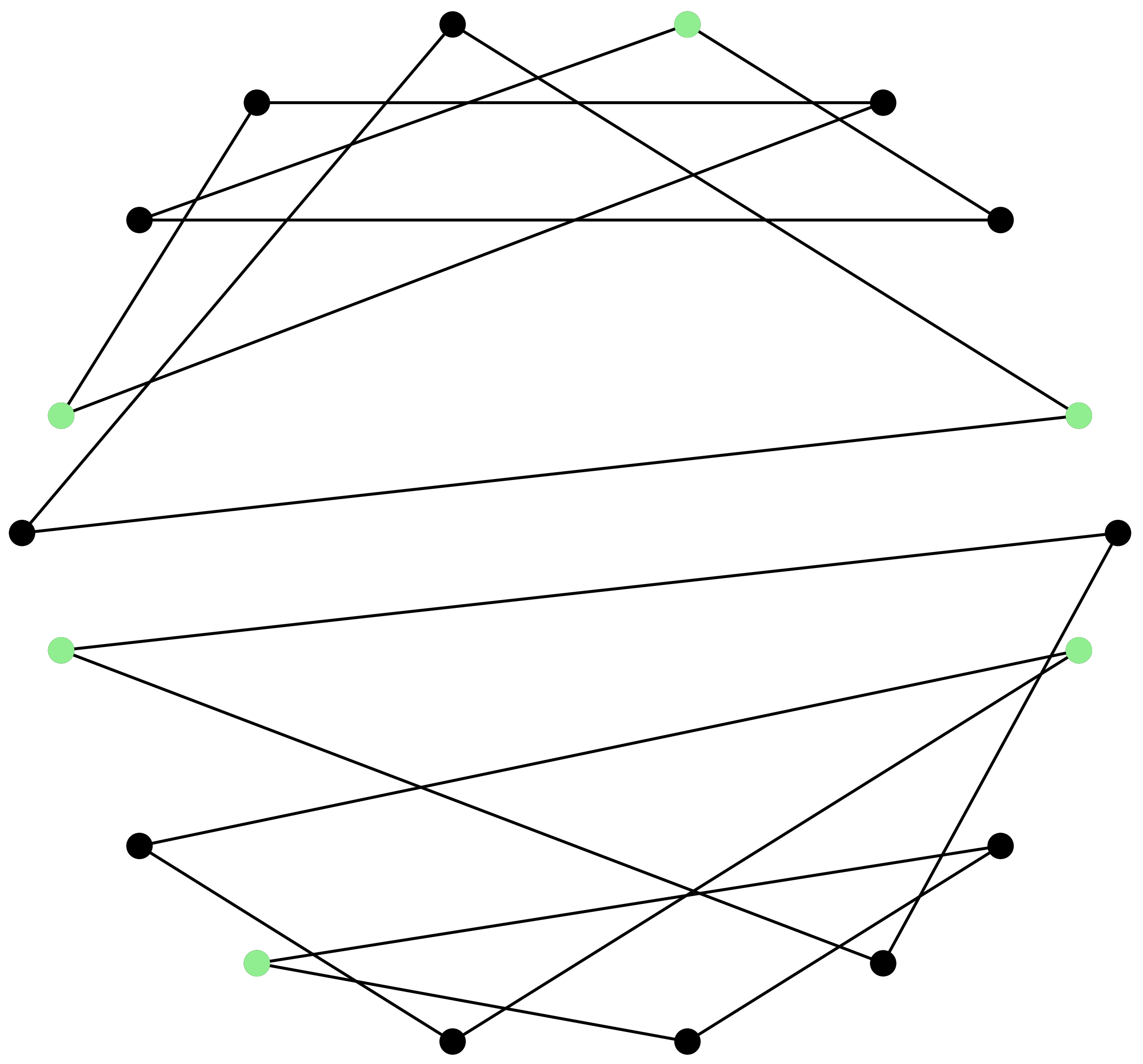

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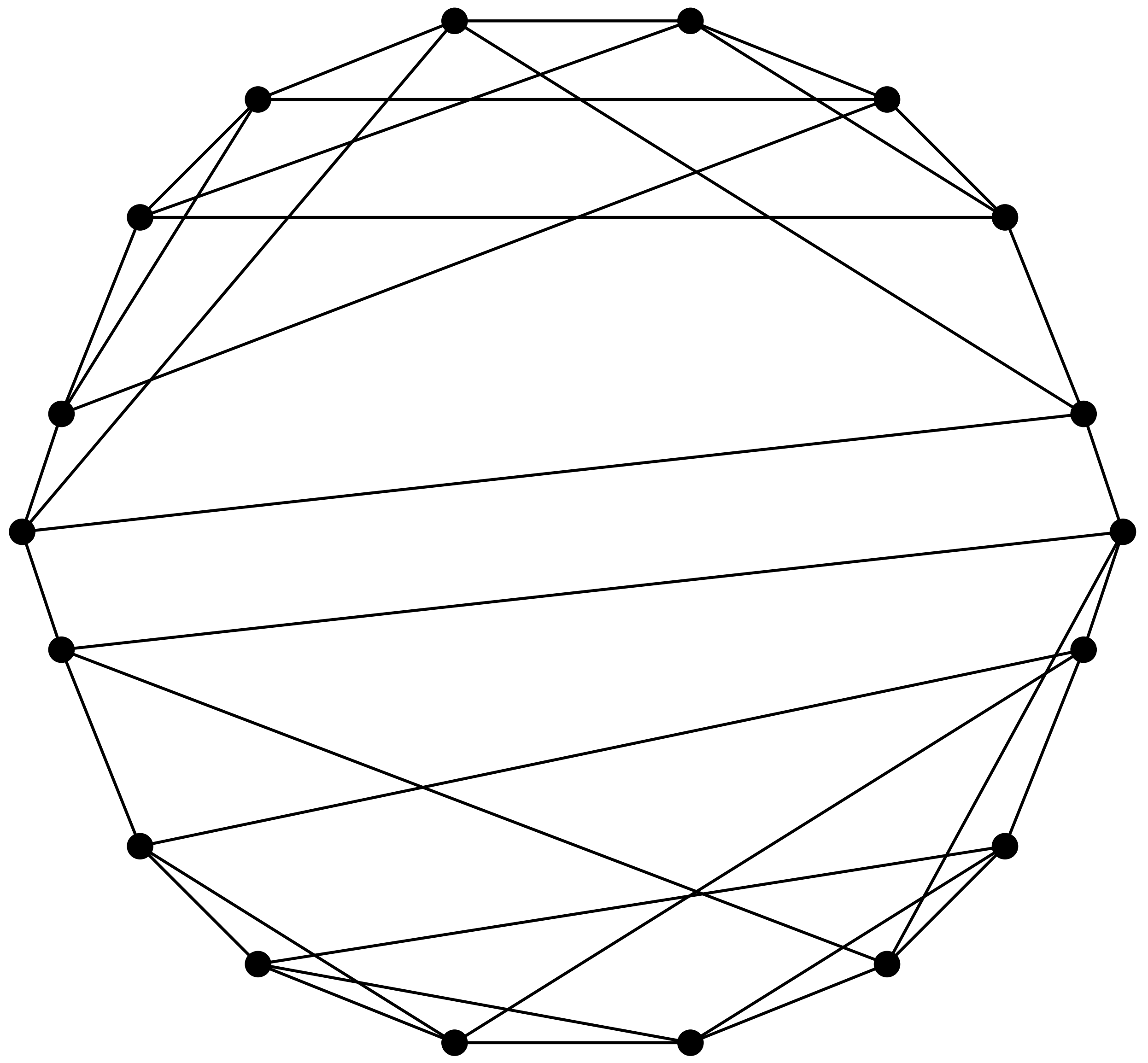

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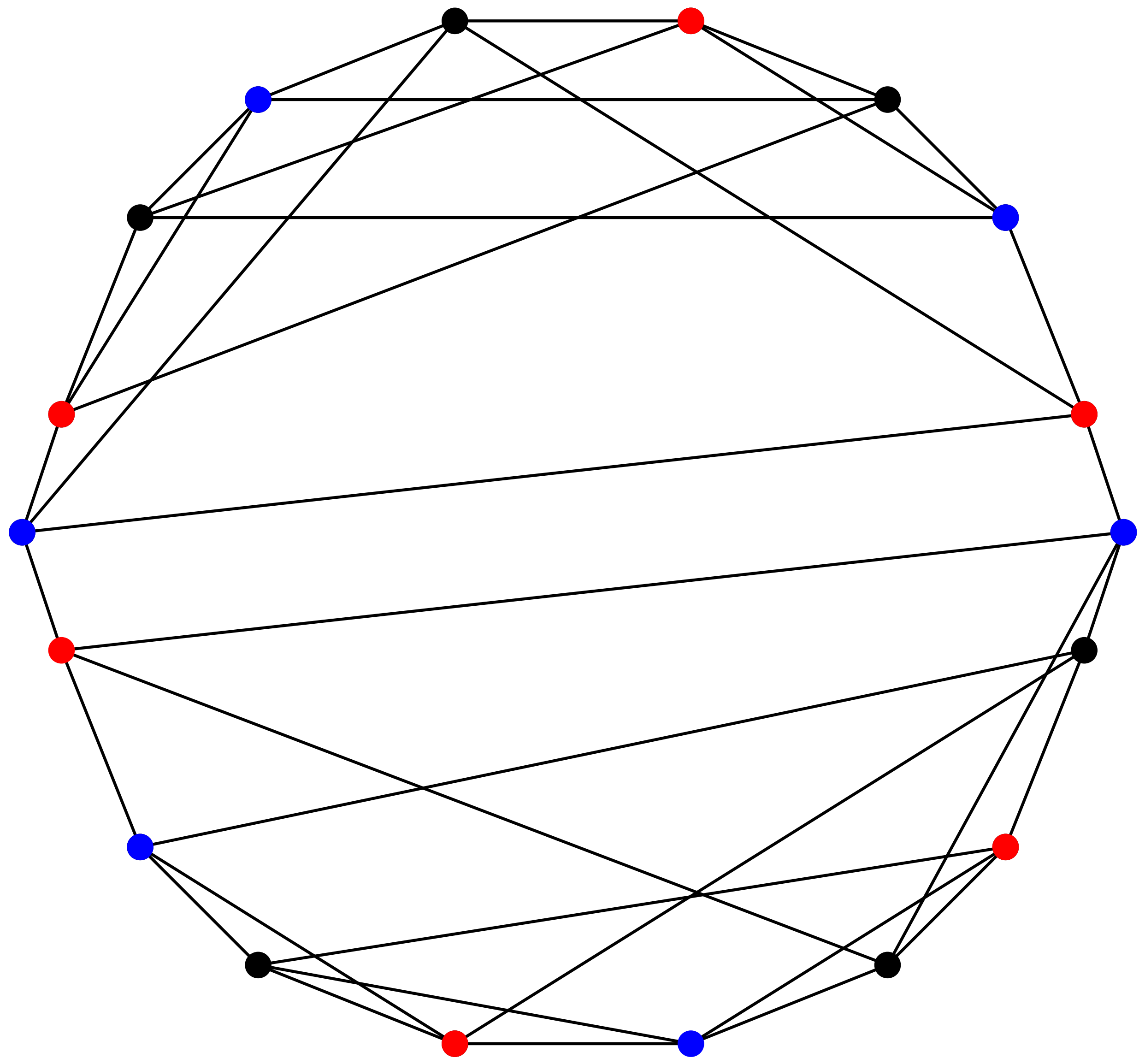

Alon-Tarsi

Theorem Let D be an orientation of a given graph G such that the number of arc-induced eulerian subdigraphs of D having an even number of arcs differs from the corresponding number of subdigraphs with an odd number of arcs. Then the list-chromatic number (and the more so the chromatic number) of G is at most $\Delta^+(D) + 1$.

References

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2. N. Kothari, M. Carvalho, C. Lucchesi, C. Little. On essentially 4-edge-connected cubic bricks.
3. L. Andersen, H. Fleischner, S. Regner. Algorithms and outerplanar conditions for A-trails in plane Eulerian graphs.