

Making the Rules of Sports Fairer

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- | Five rounds are played, in each round first a player from the first team, and then from the other, attempts to score.
- | If after five rounds scores are tied, the game goes to sudden death.



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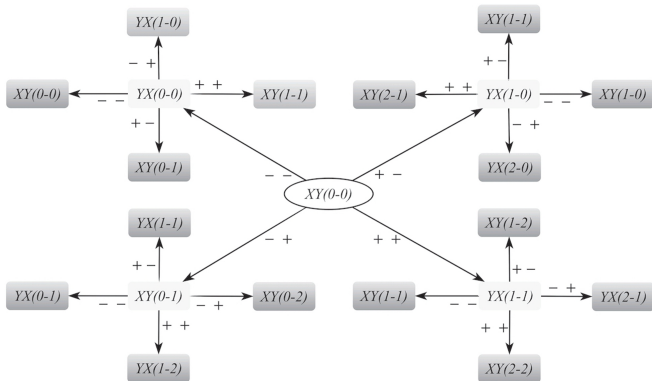
In a survey more than 90% of players and coaches said they would choose to go first. No one chose going second.

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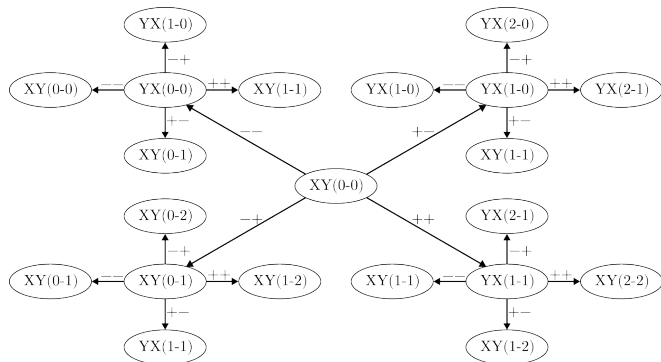


Behind First, Alternating Order Rule

Depends on the current score: if one team is behind, it kicks first on the next round; if the score is tied, the order of kicking alternates.

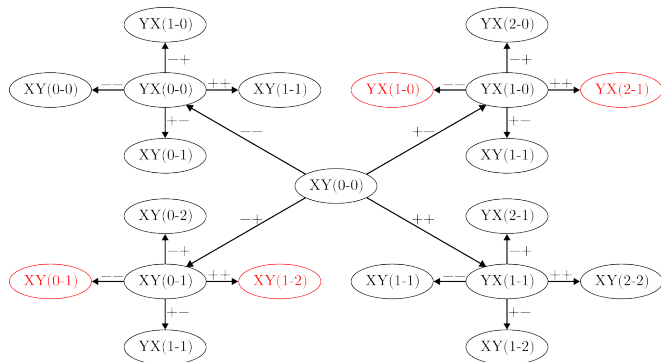
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- | q - probability of a team scoring when it kicks second

Example: $\rho = \frac{3}{4}$, $q = \frac{2}{3}$

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Standard Rule

$$P^2(X) = \frac{51}{144} = 0.354$$

$$P^2(Y) = \frac{32}{144} = 0.222$$

$$P^2(T) = \frac{61}{144} = 0.424$$

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Catch-Up Rule

$$P^2(X) = \frac{41}{144} = 0.284$$

$$P^2(Y) = \frac{39}{144} = 0.270$$

$$P^2(T) = \frac{64}{144} = 0.444$$

Comparison on more rounds

Assuming empirical probabilities:

$$[p_i; q_i]_{i=1}^5 = [(0:79; 0:72); (0:82; 0:77); (0:77; 0:64); \\ (0:74; 0:68); (0:74; 0:67)]$$

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Resulting winning probabilities after 1-5 rounds:

Comparison when one team is better skilled

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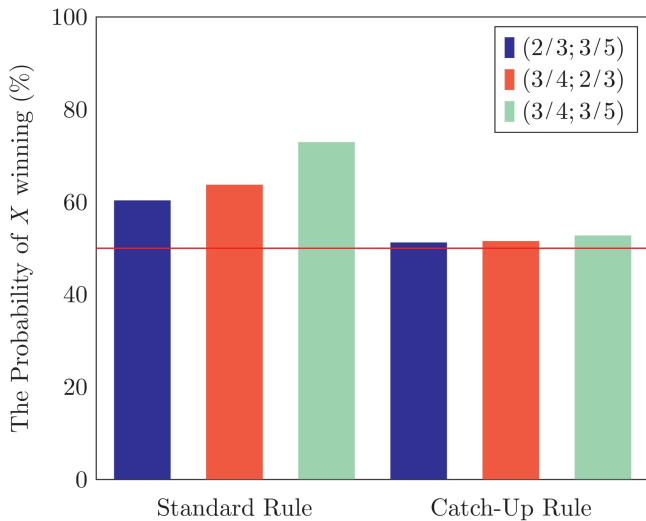
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$$(1 + q - p) - (p - q) = 1 - 2(p - q)$$

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References

- | Steven J. Brams, Mehmet S. Ismail, *Making the Rules of Sports Fairer*, 2018

Clip art sources

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- | flyclipart.com: [1], [2]
- | openclipart.org: [1]
- | www.clipproject.info: [1]