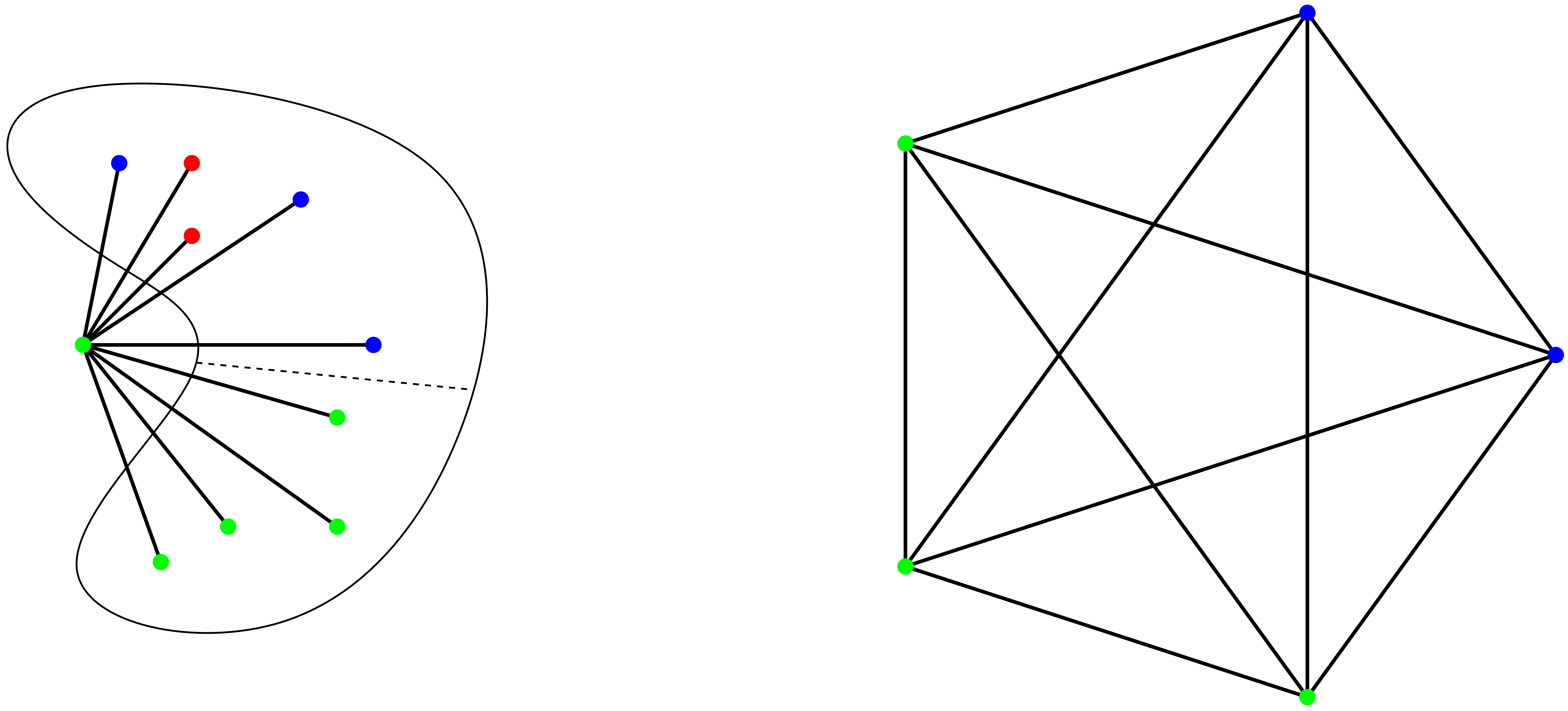
Countable graphs are majority 3-choosable

John Haslegrave

Optymalizacja Kombinatoryczna 2023/24Z

Majority coloring

A coloring, in which at most half of the edges adjacent to each vertex are monochromatic.



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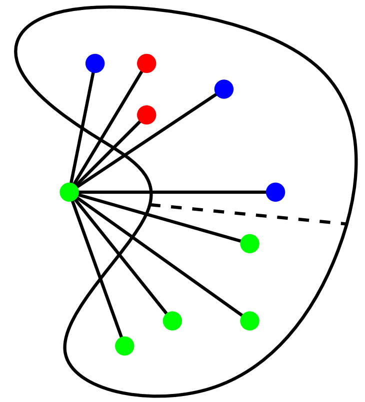

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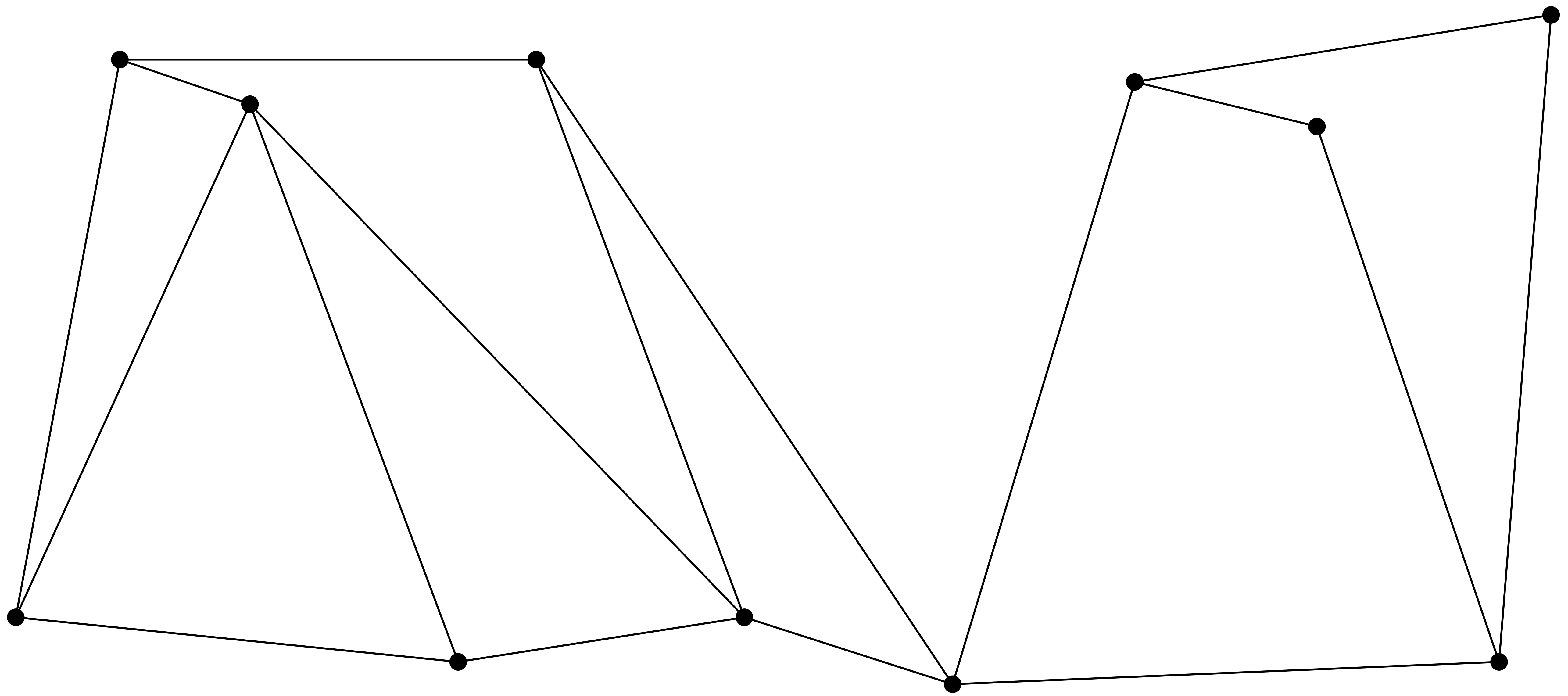
Is every finite graph majority $2-$colorable?



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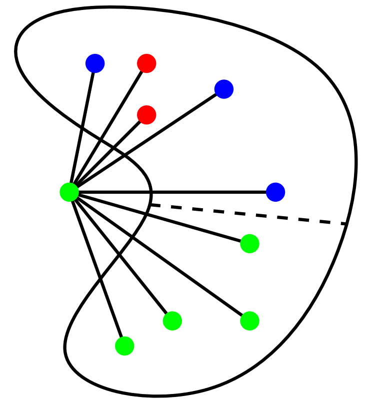

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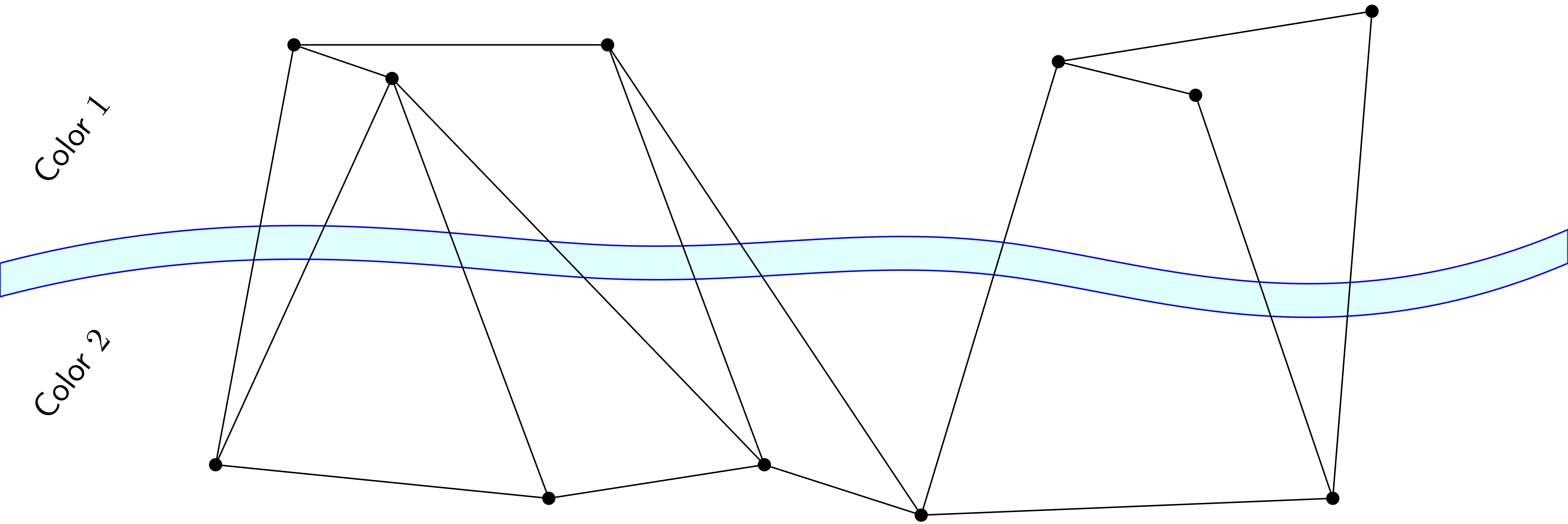

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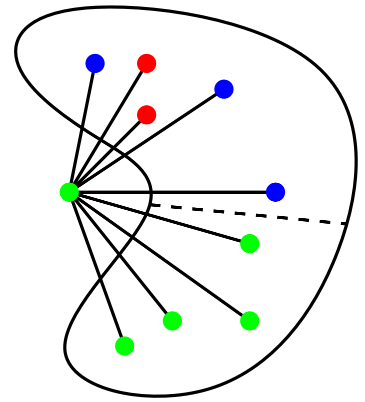

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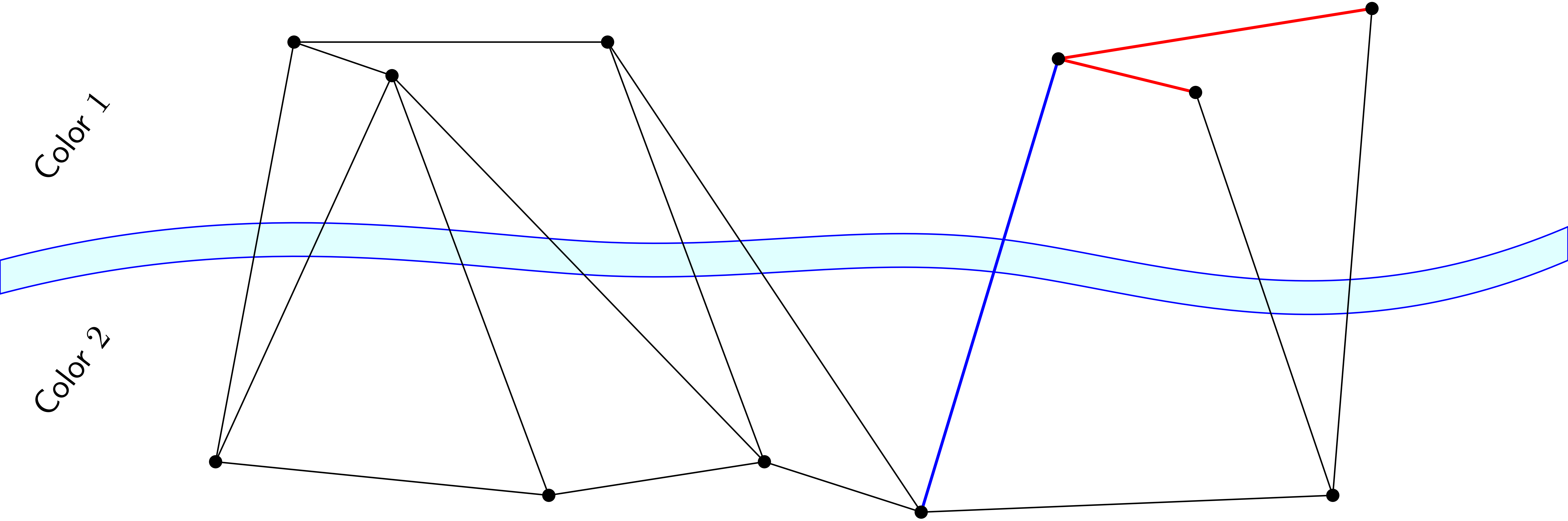

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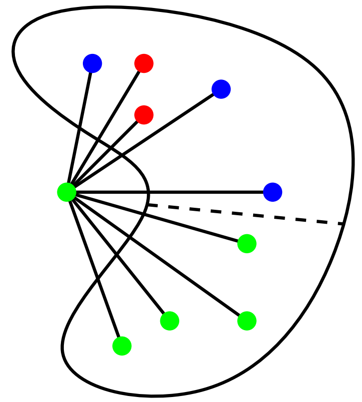

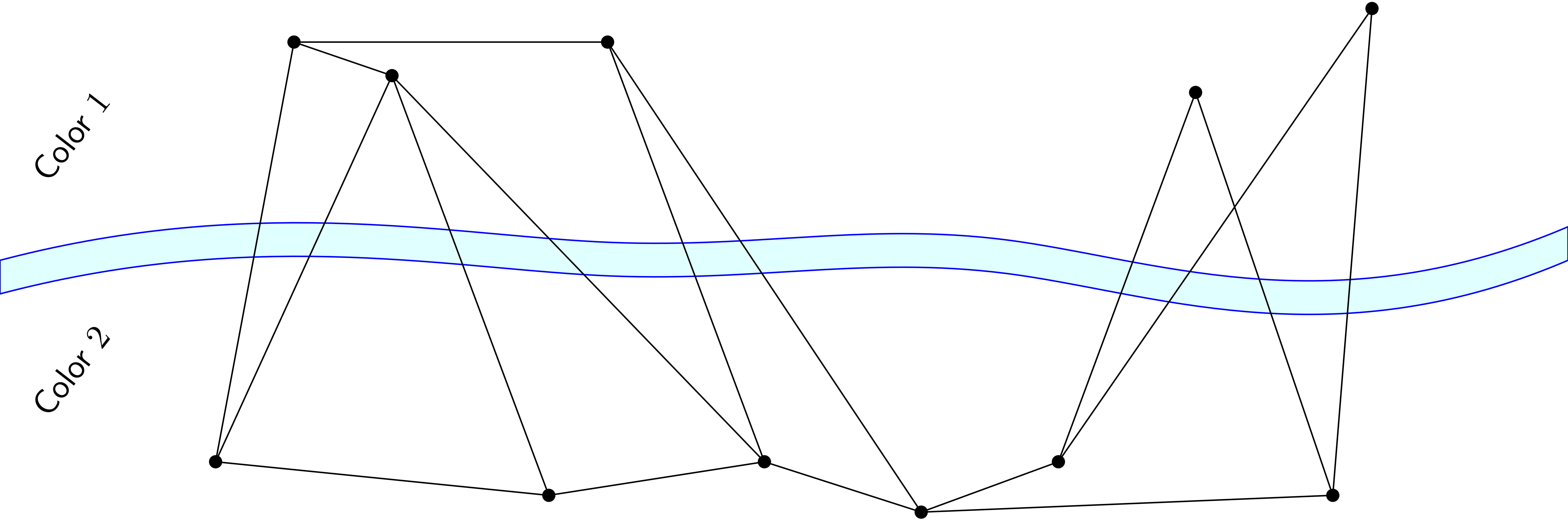

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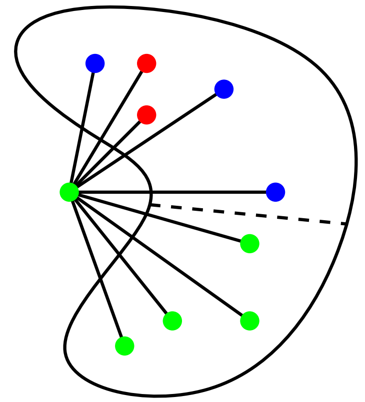

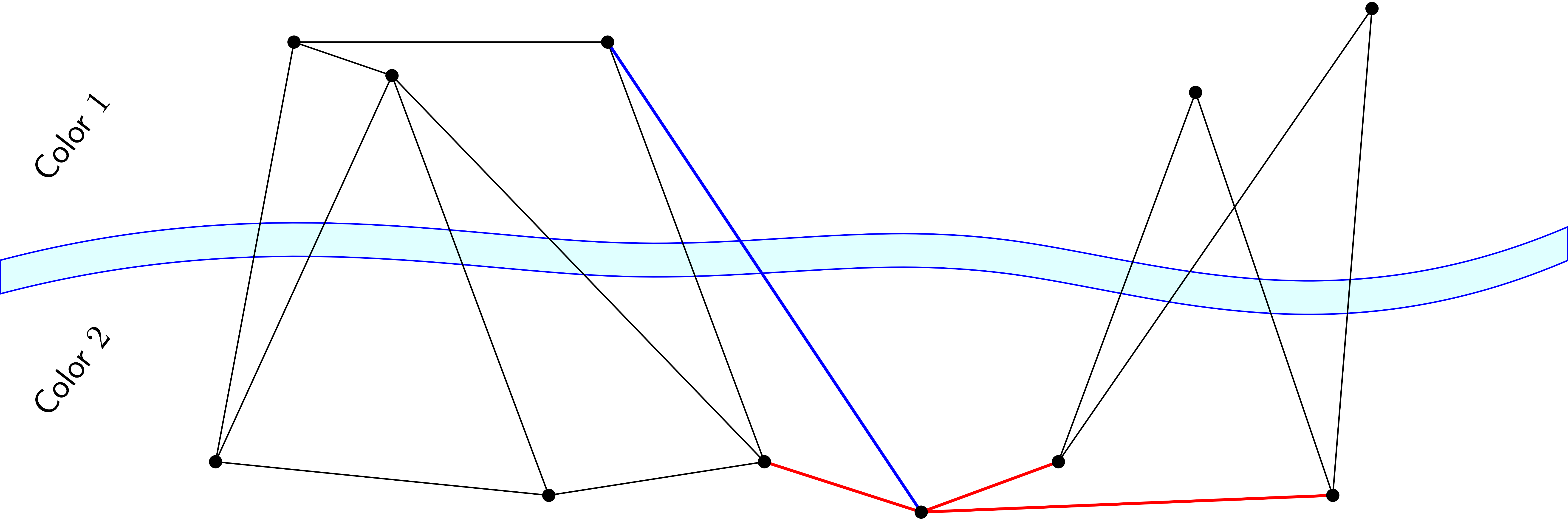

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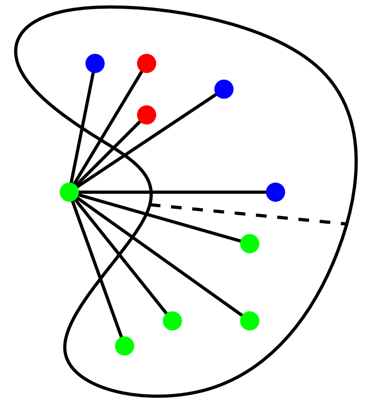

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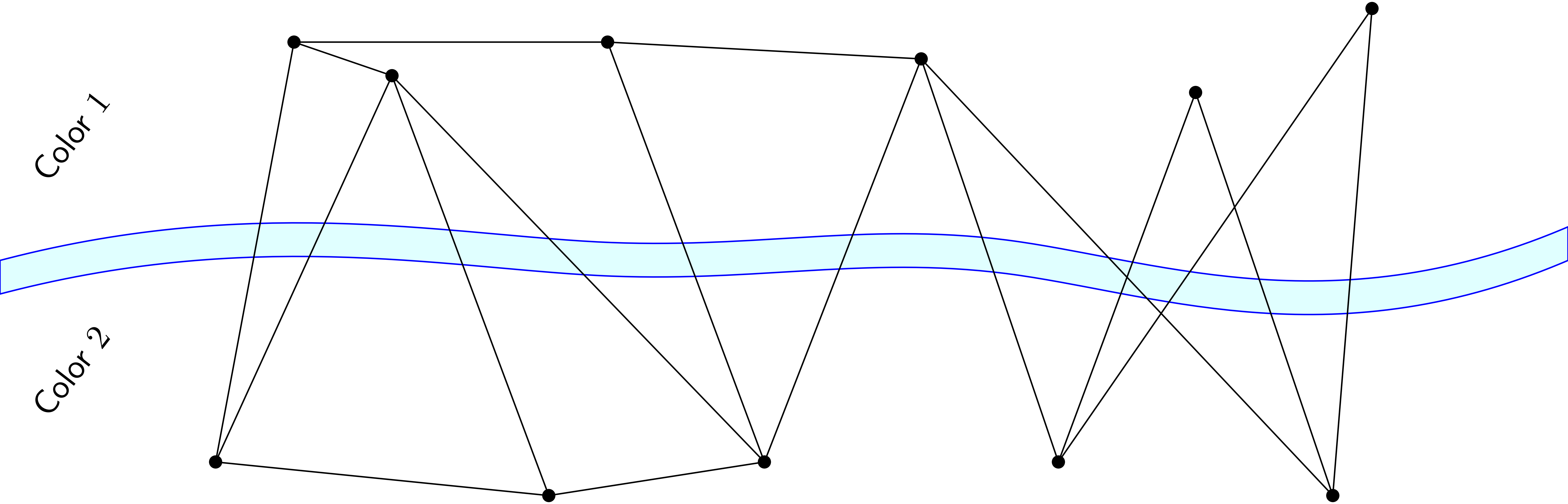

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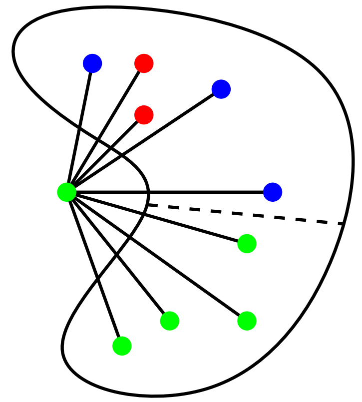

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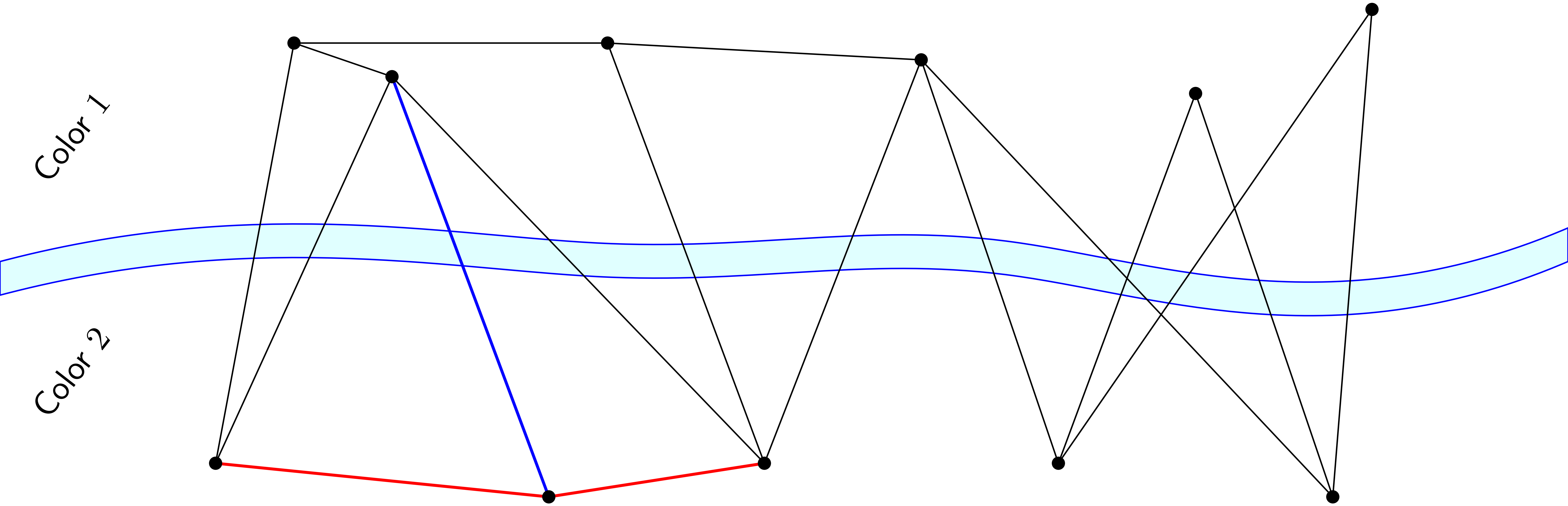

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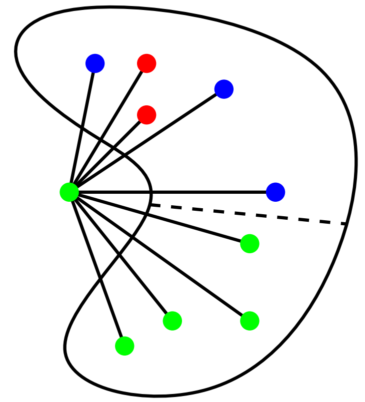

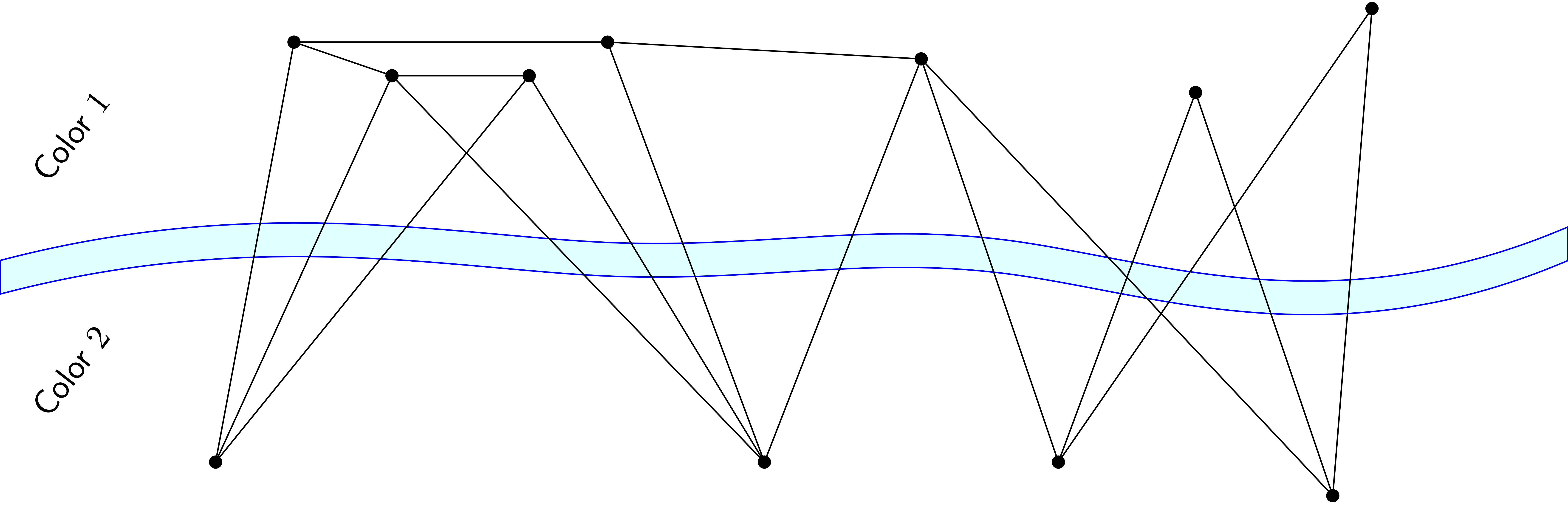

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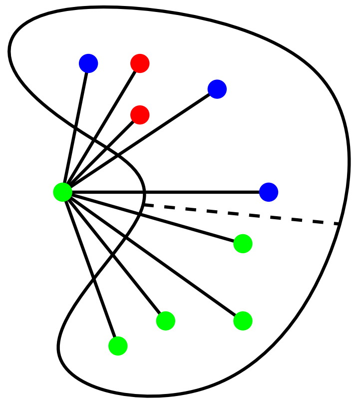

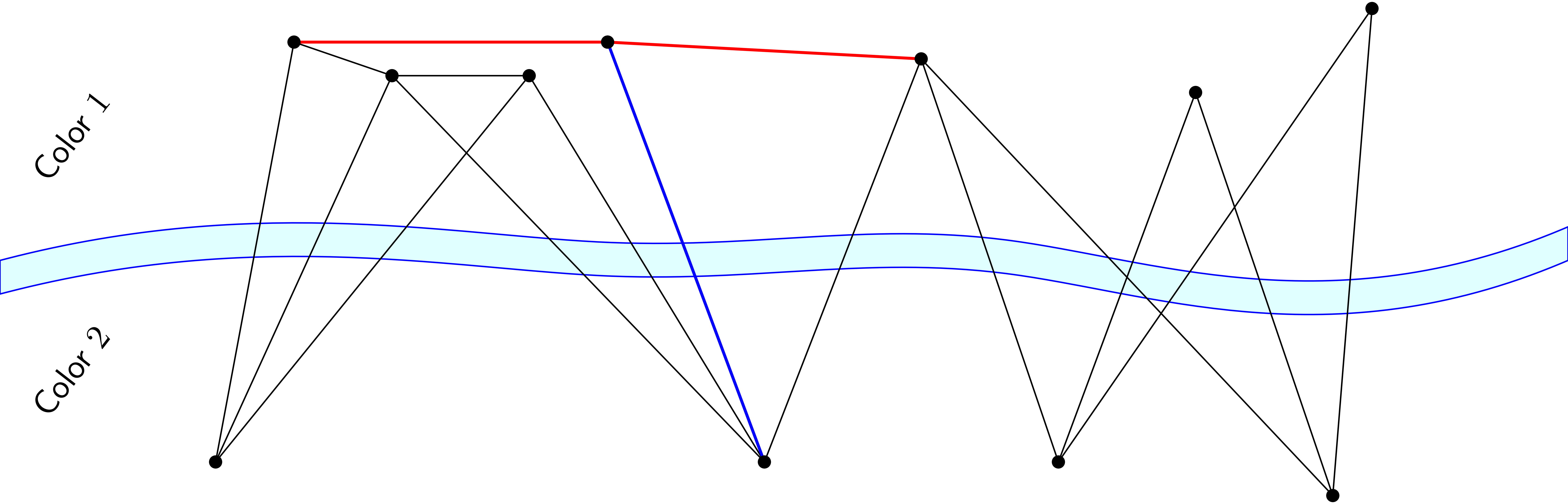

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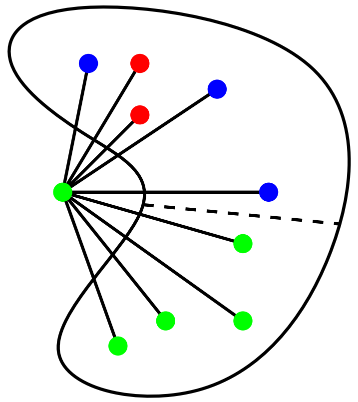

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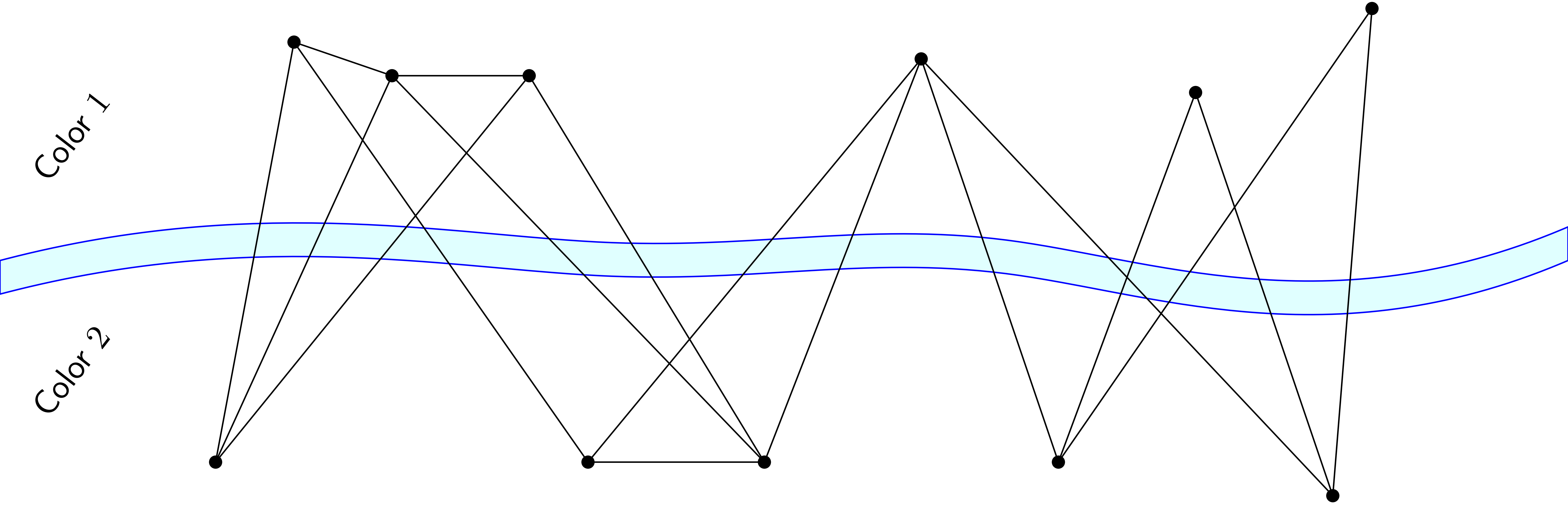

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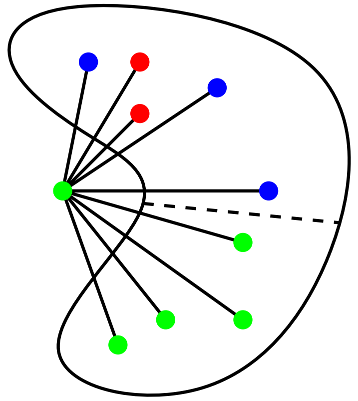

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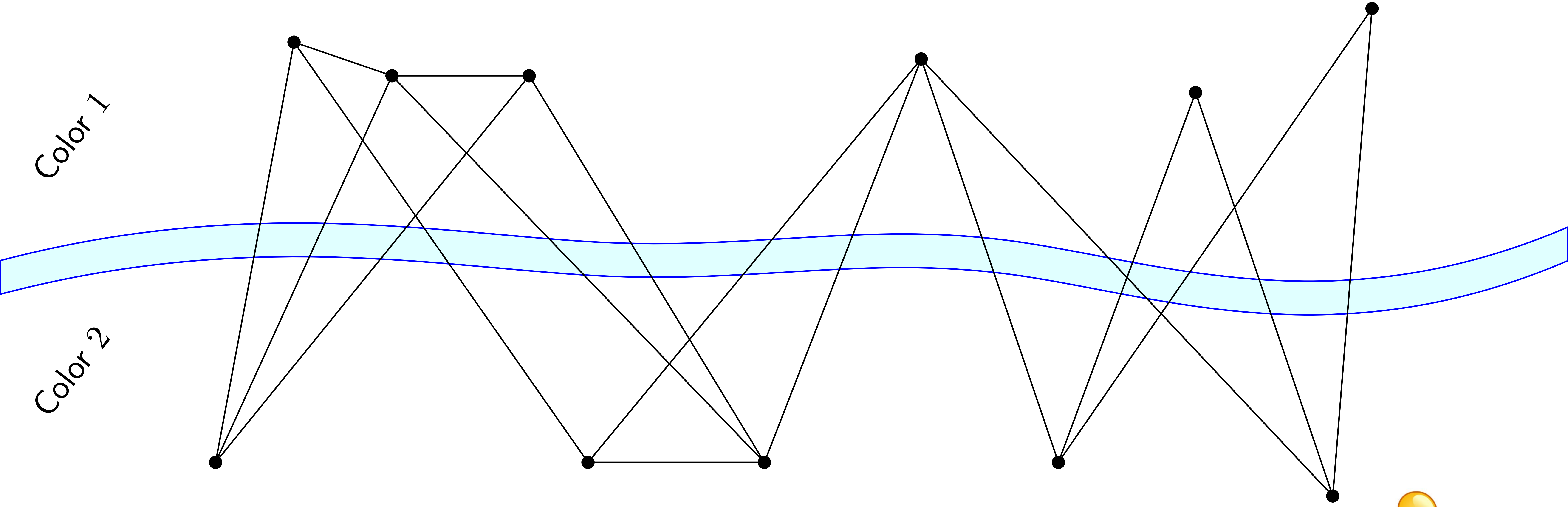

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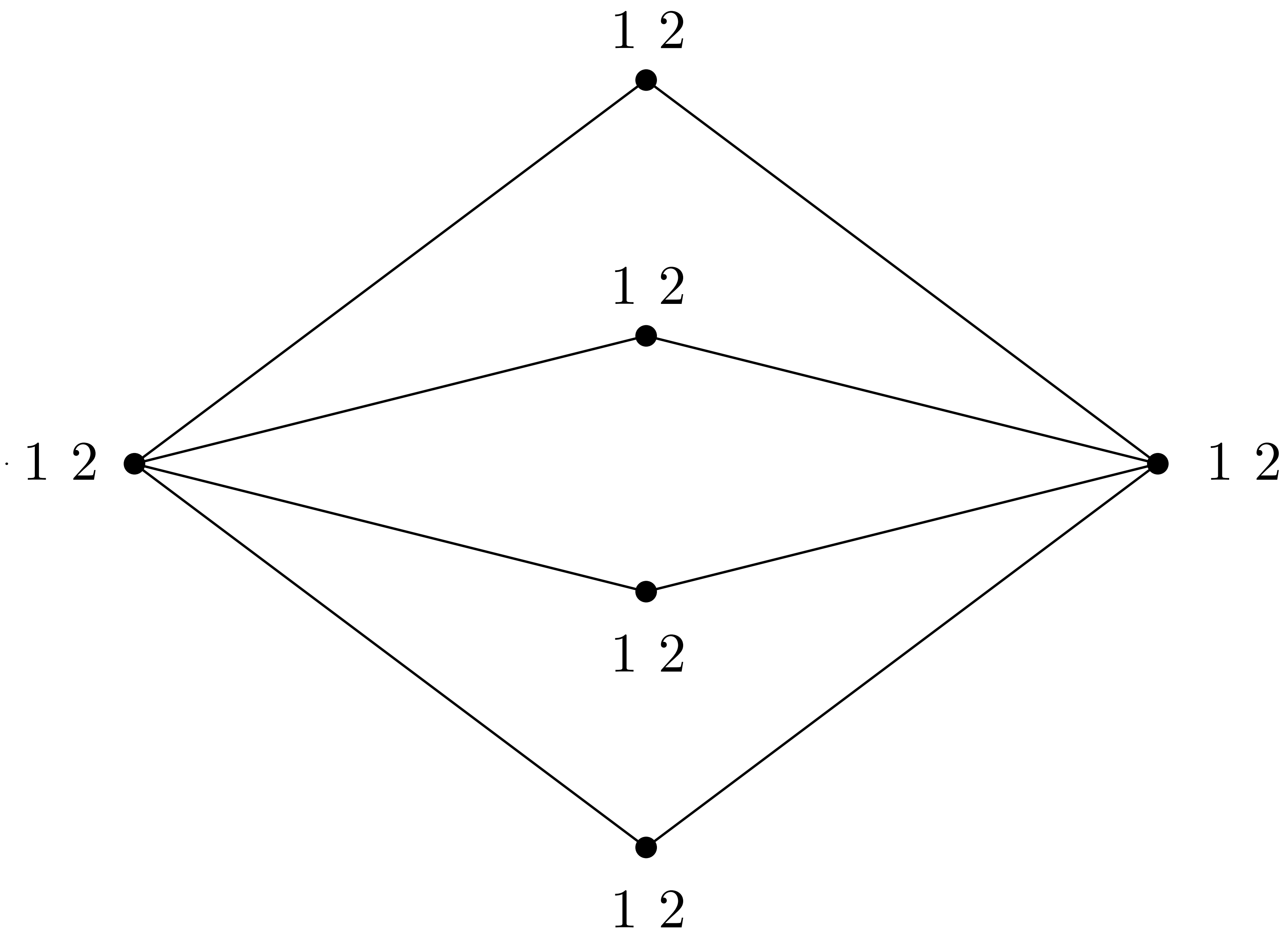
Instead of coloring vertices with 1 and 2...



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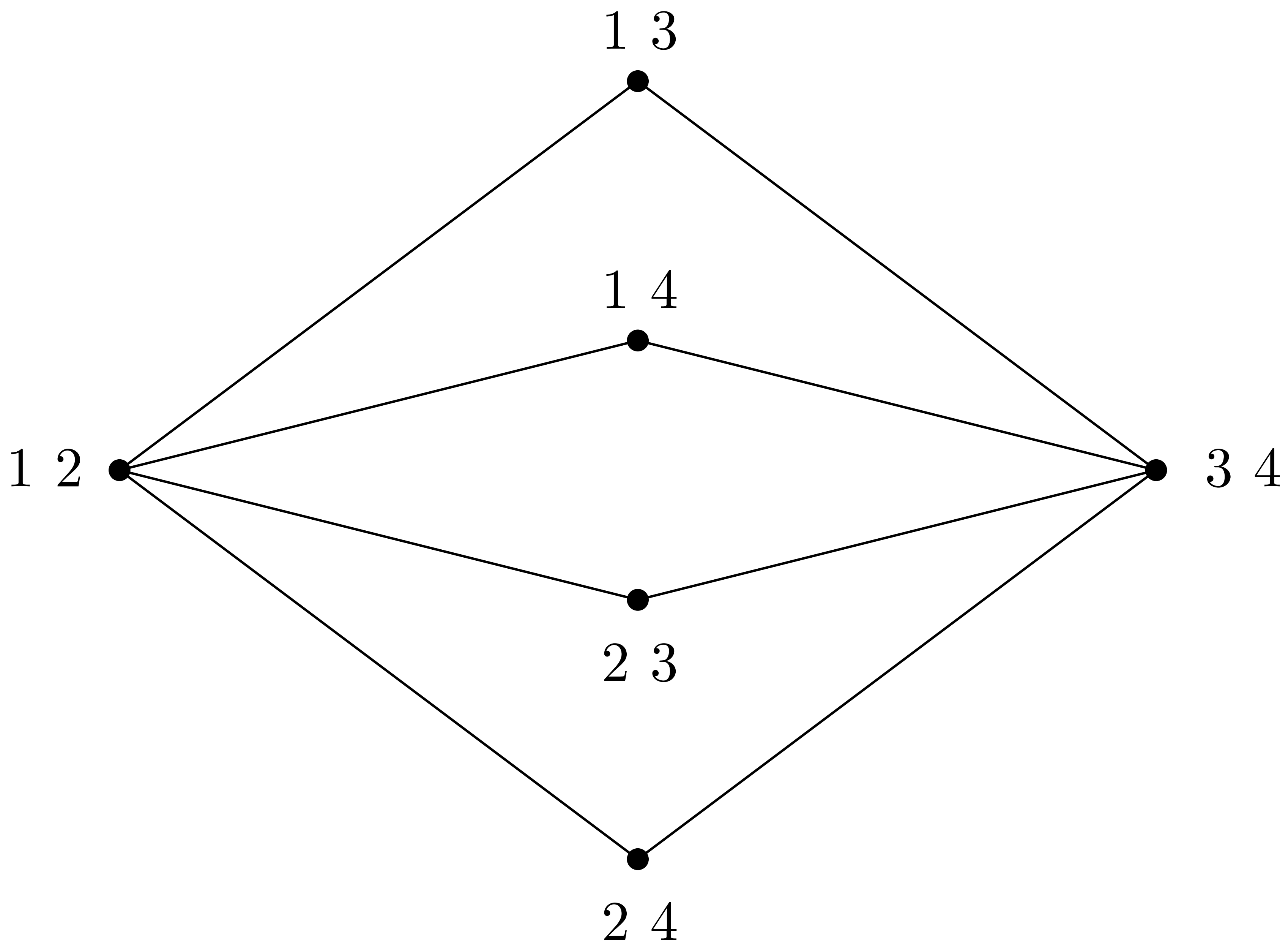

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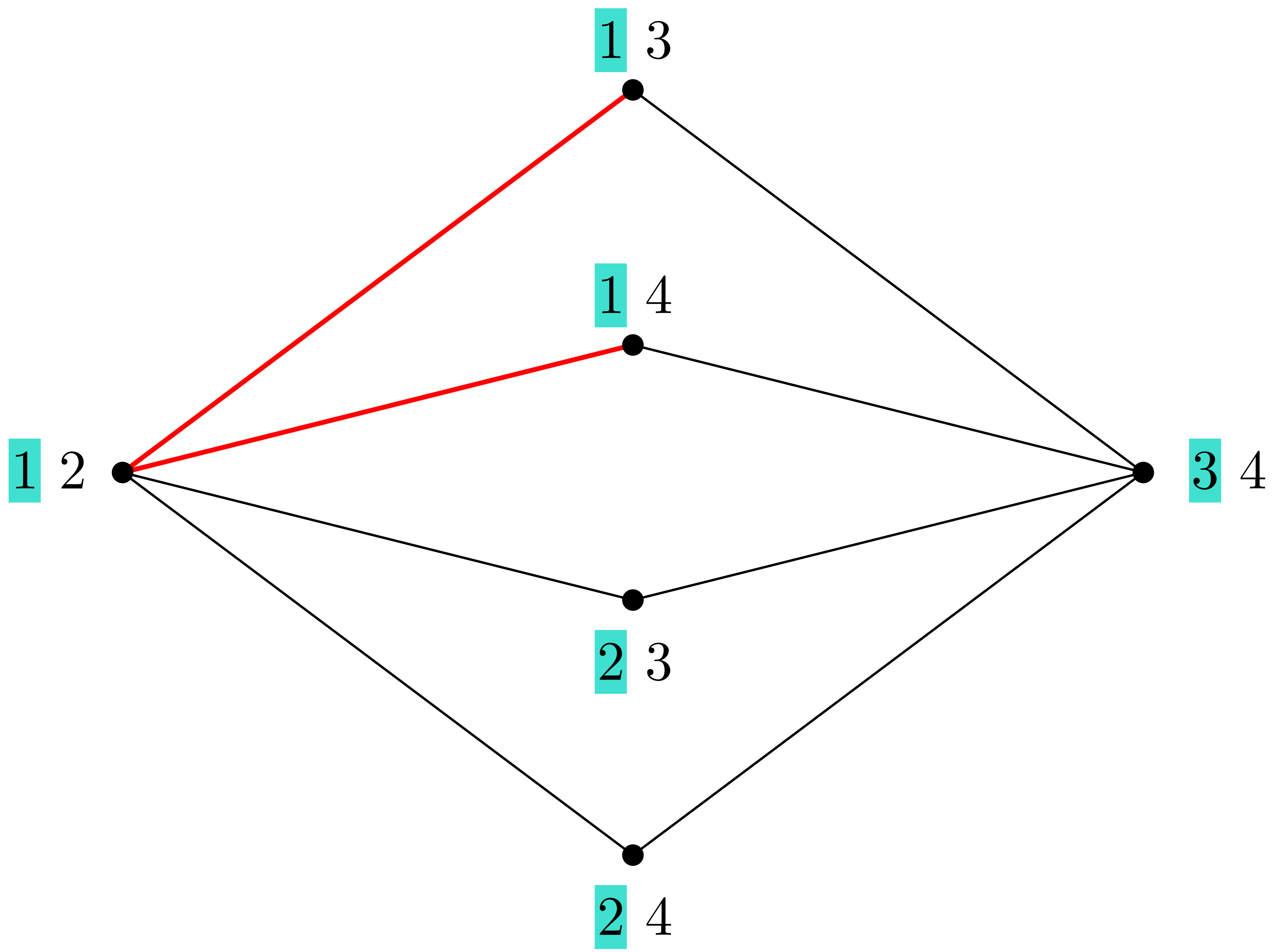

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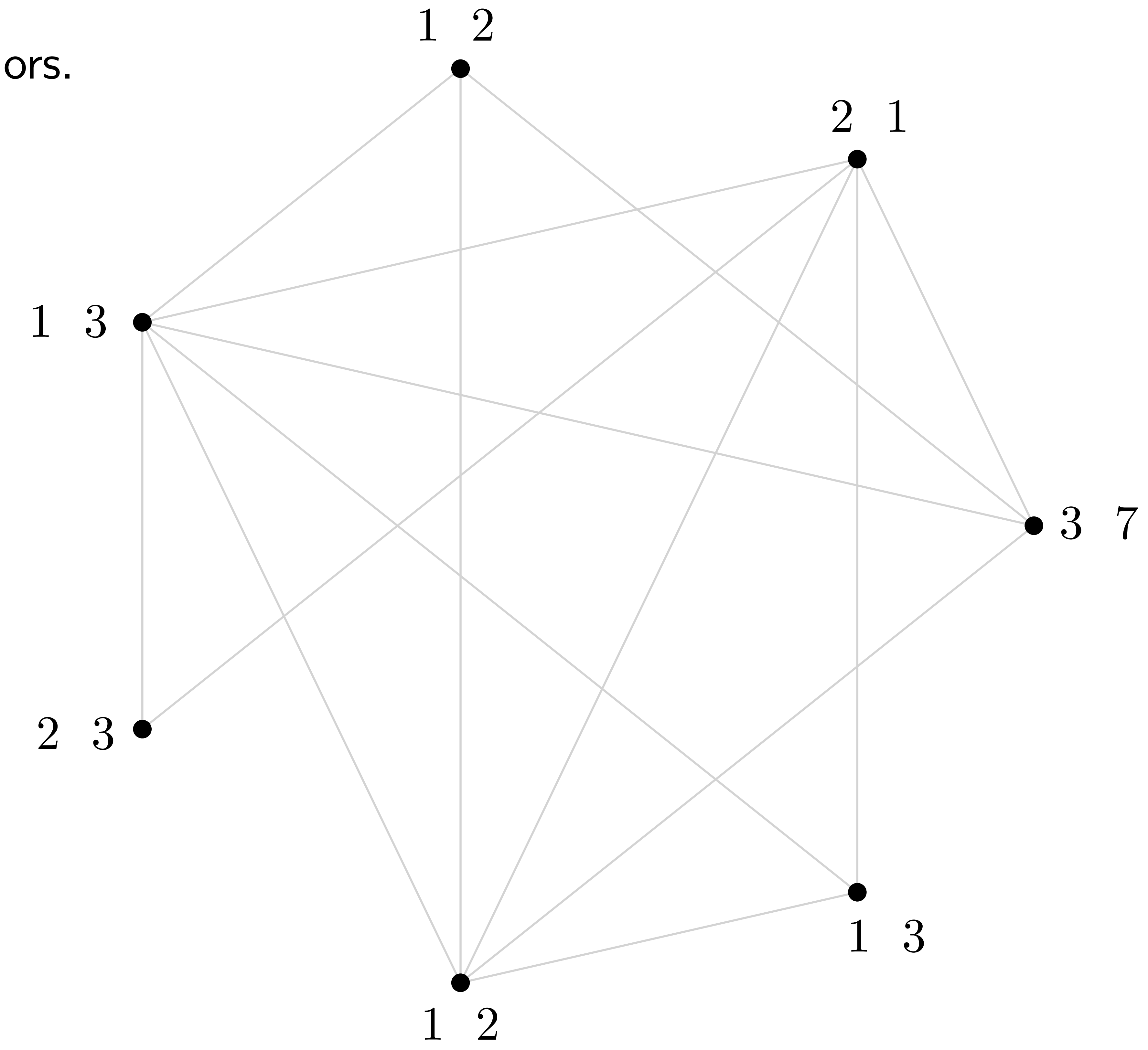

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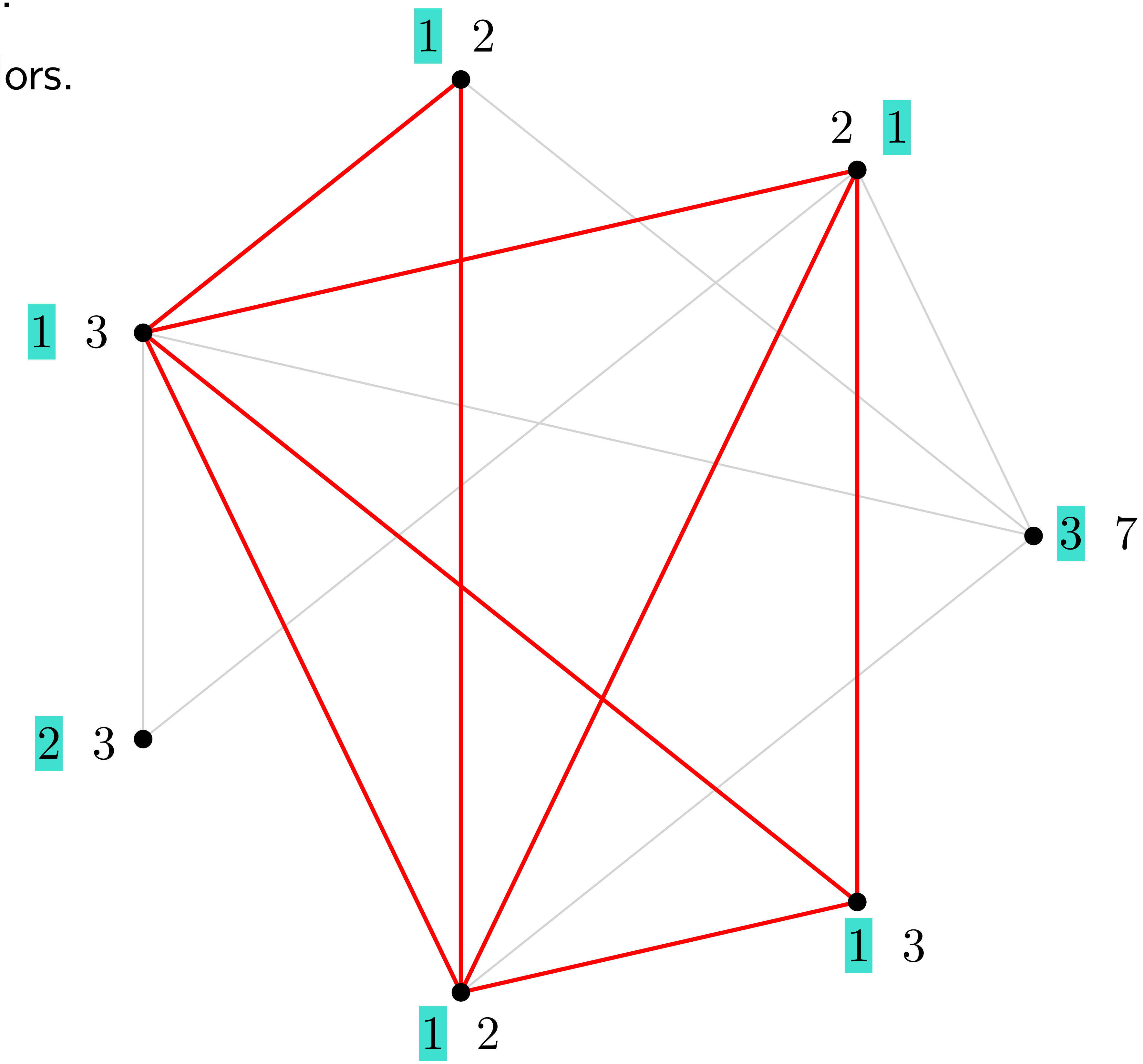

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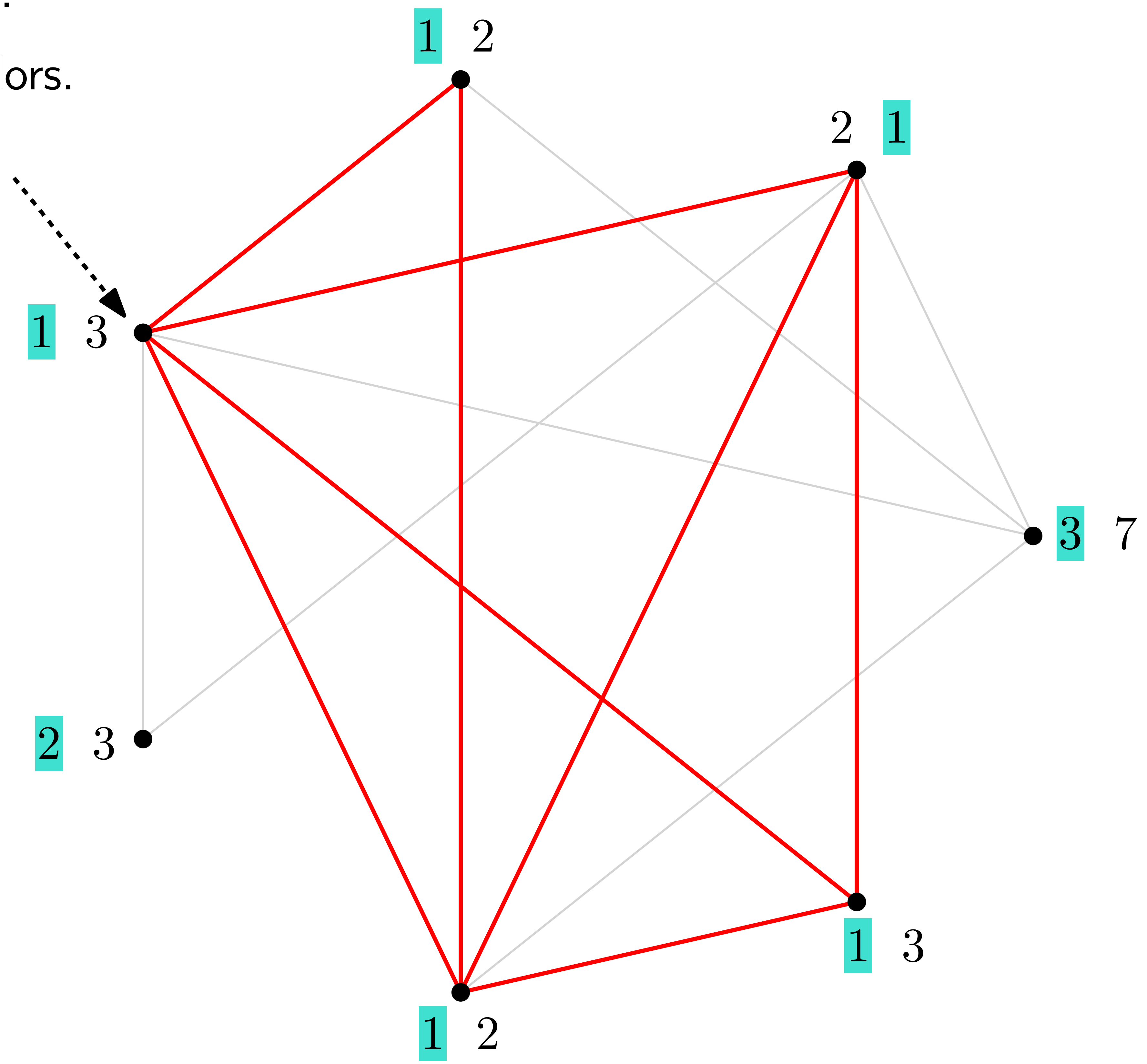

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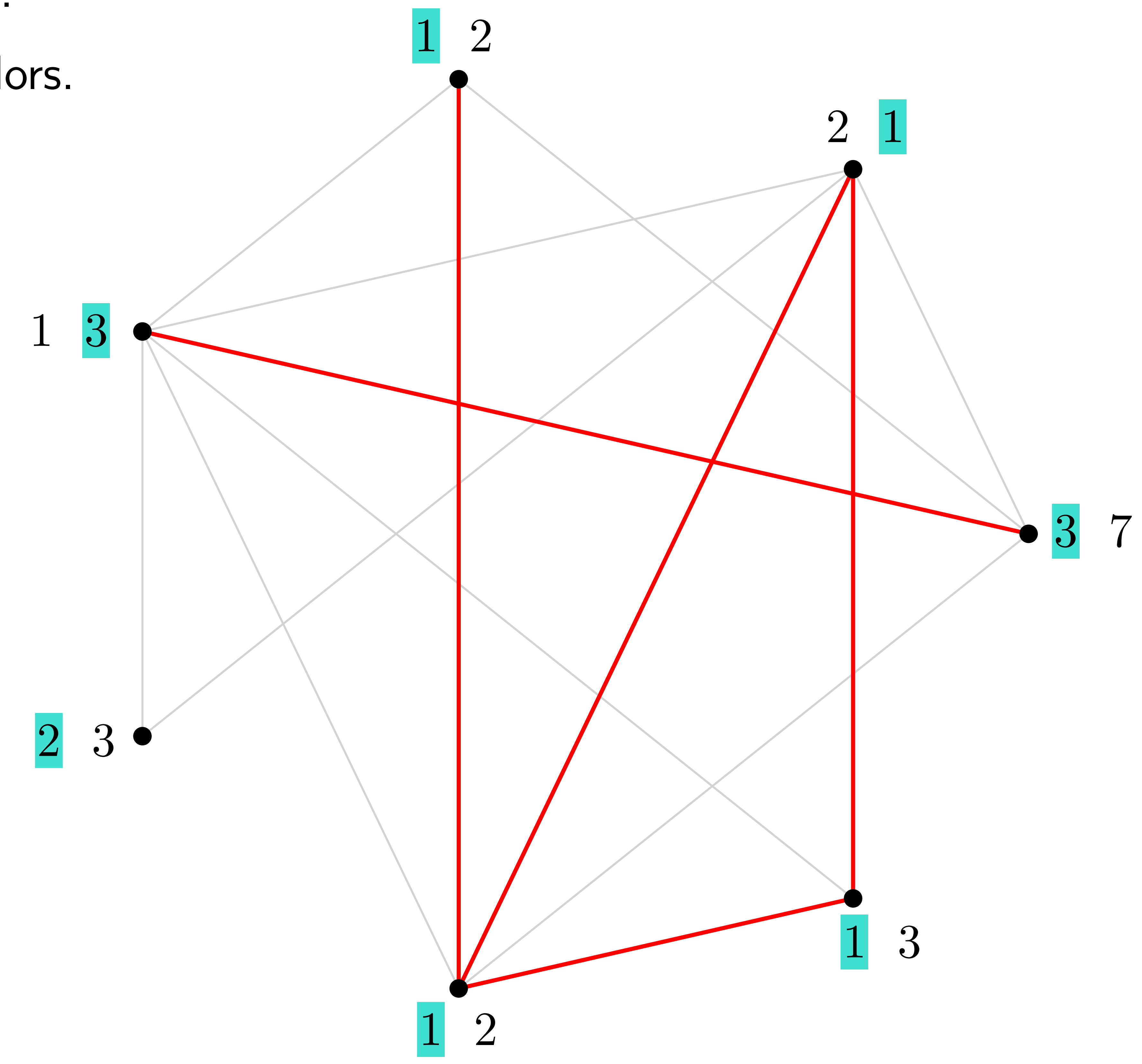

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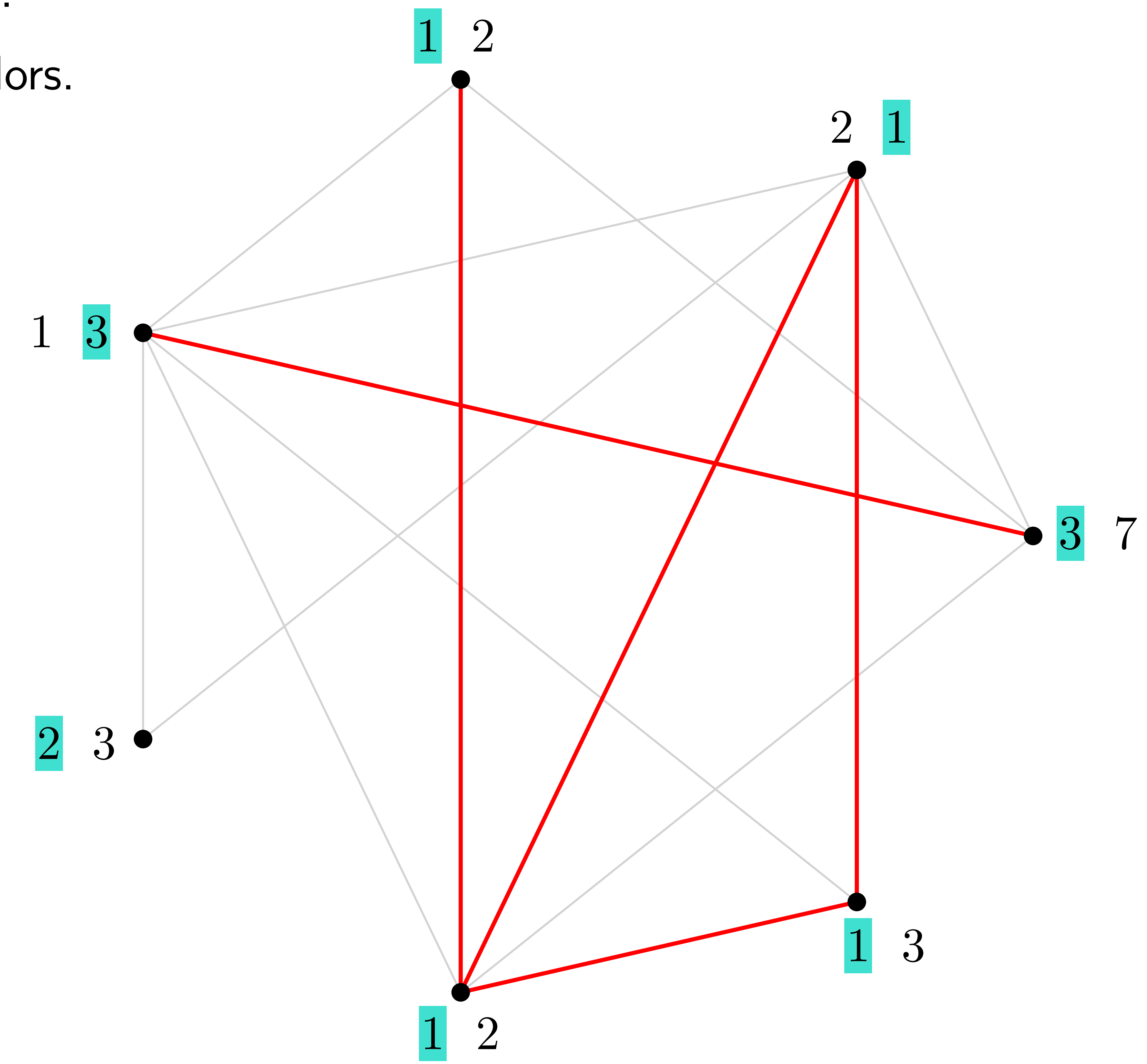

Every finite graph
is majority $2-$choosable!

Infinite graphs

Definition

A *graph* is a pair $G = (V, E)$,
where V is a set whose elements are called vertices,
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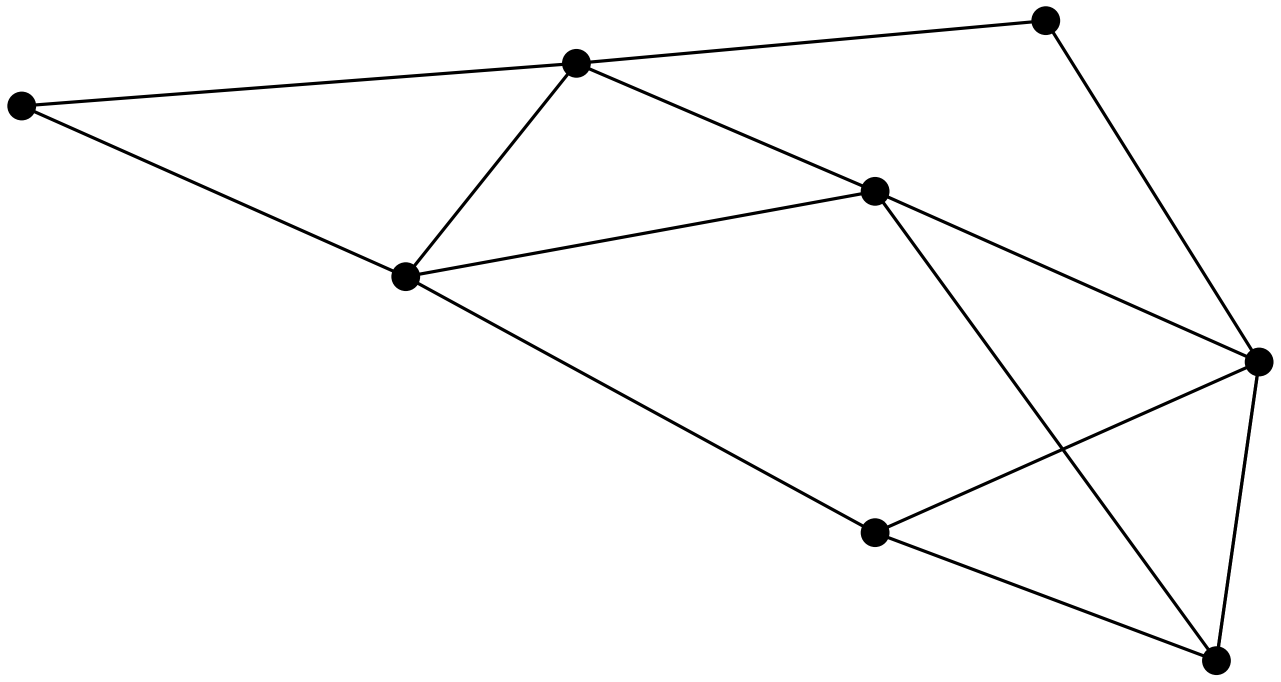

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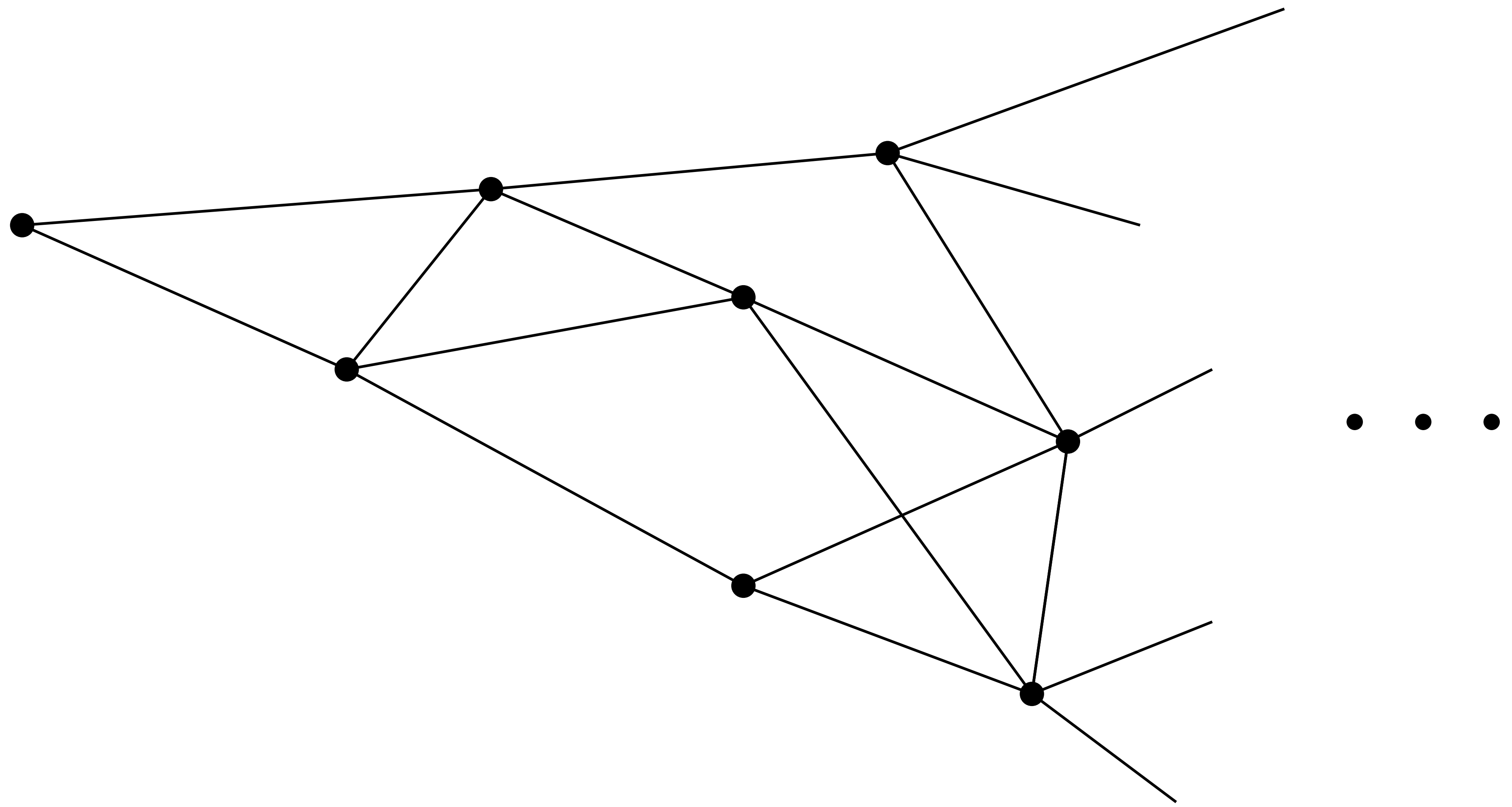

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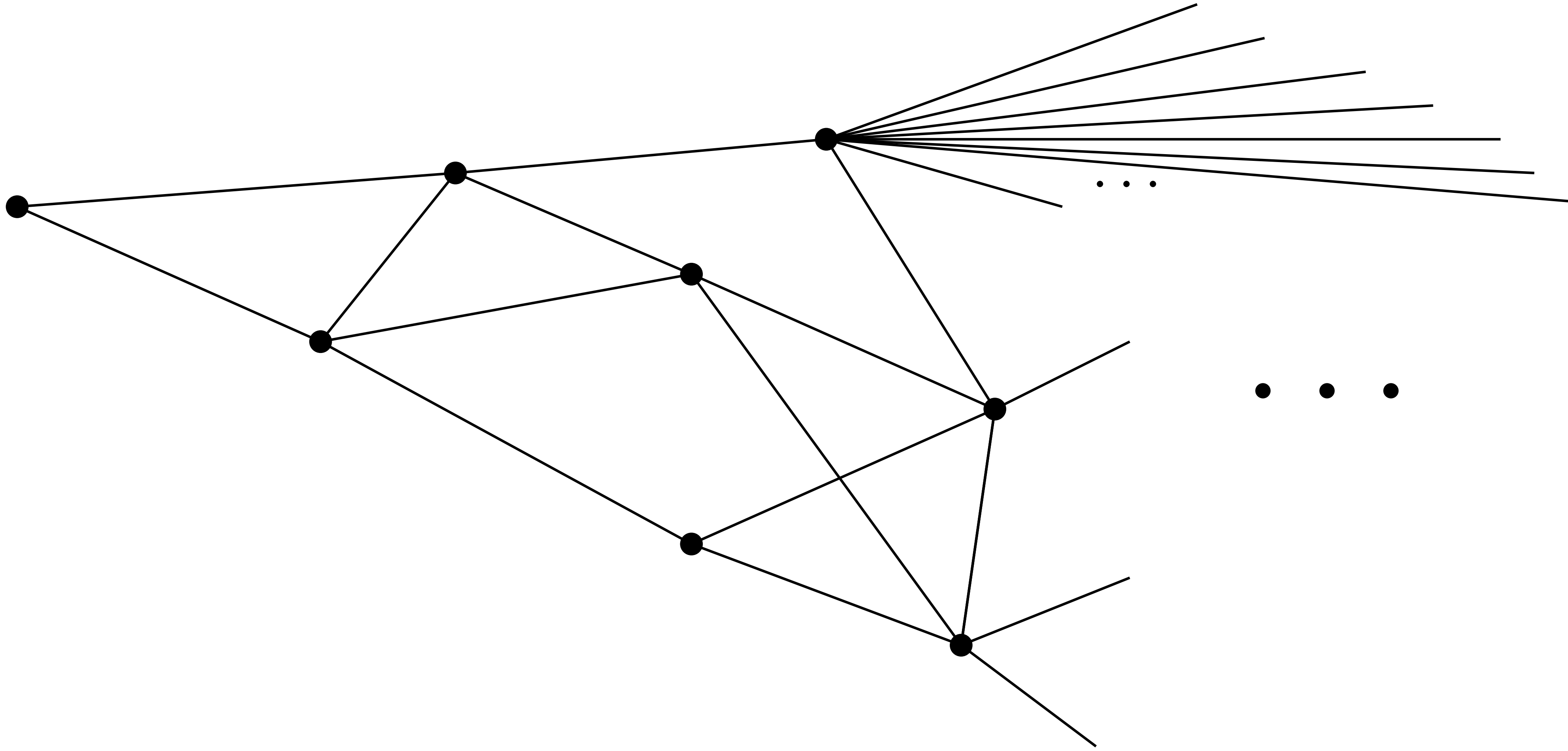

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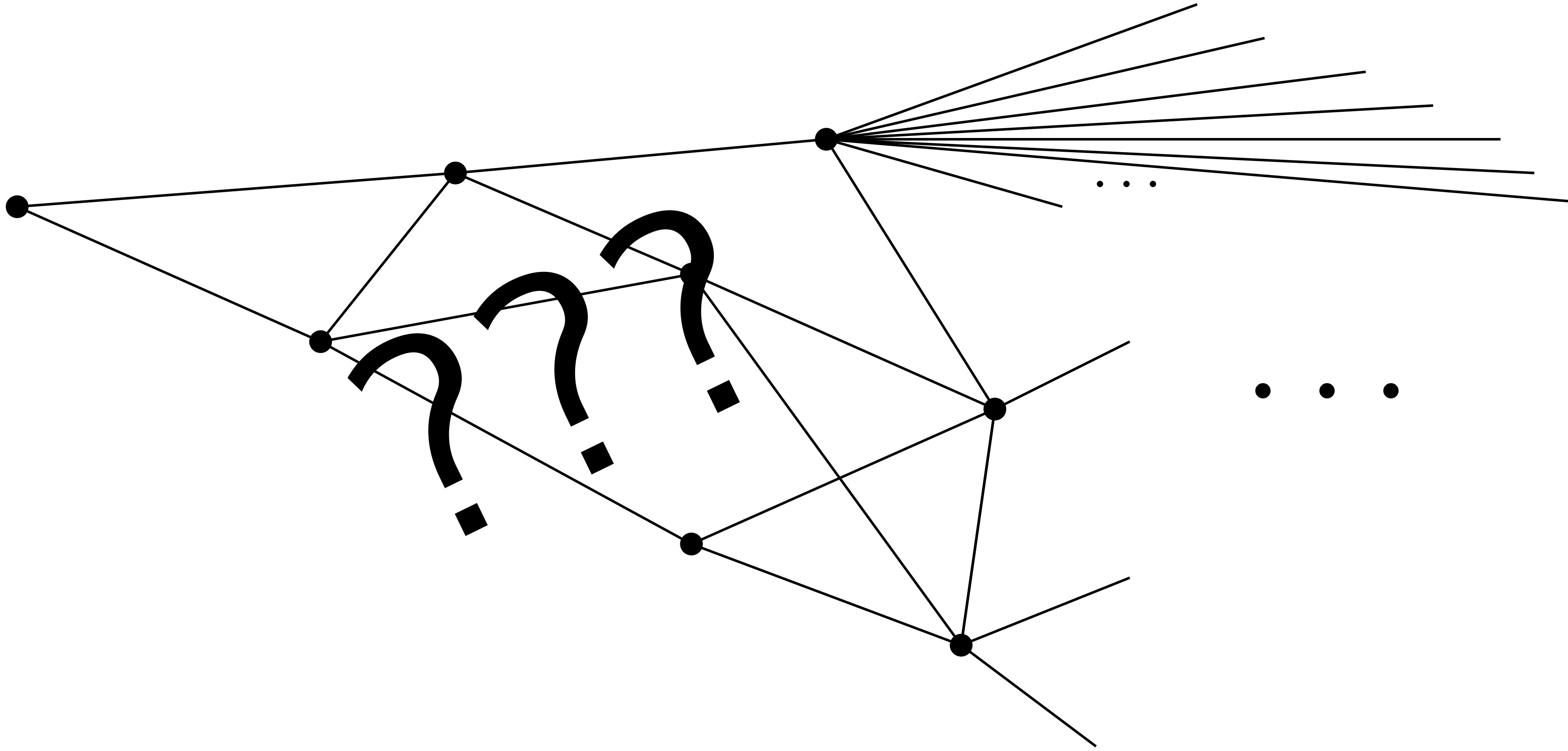

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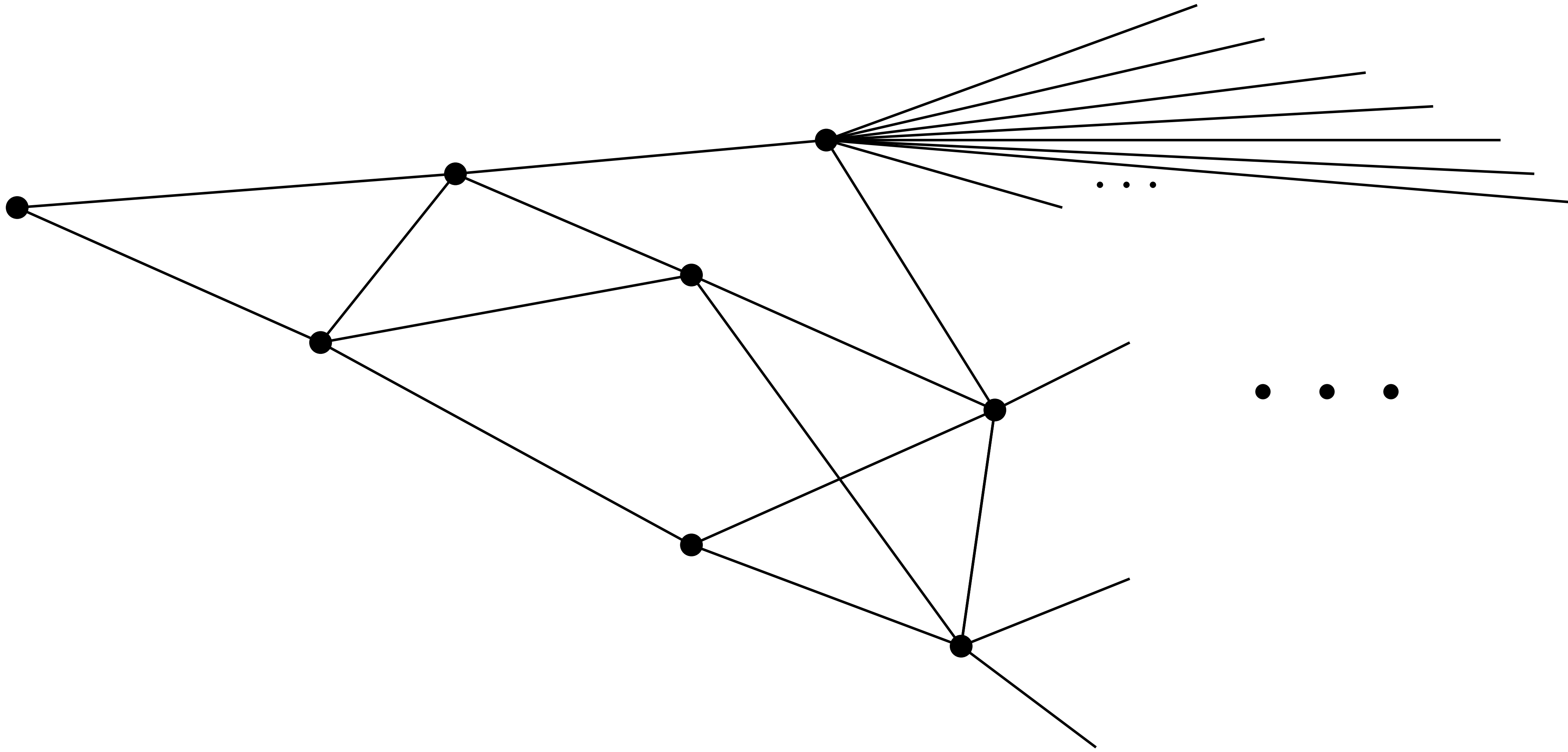

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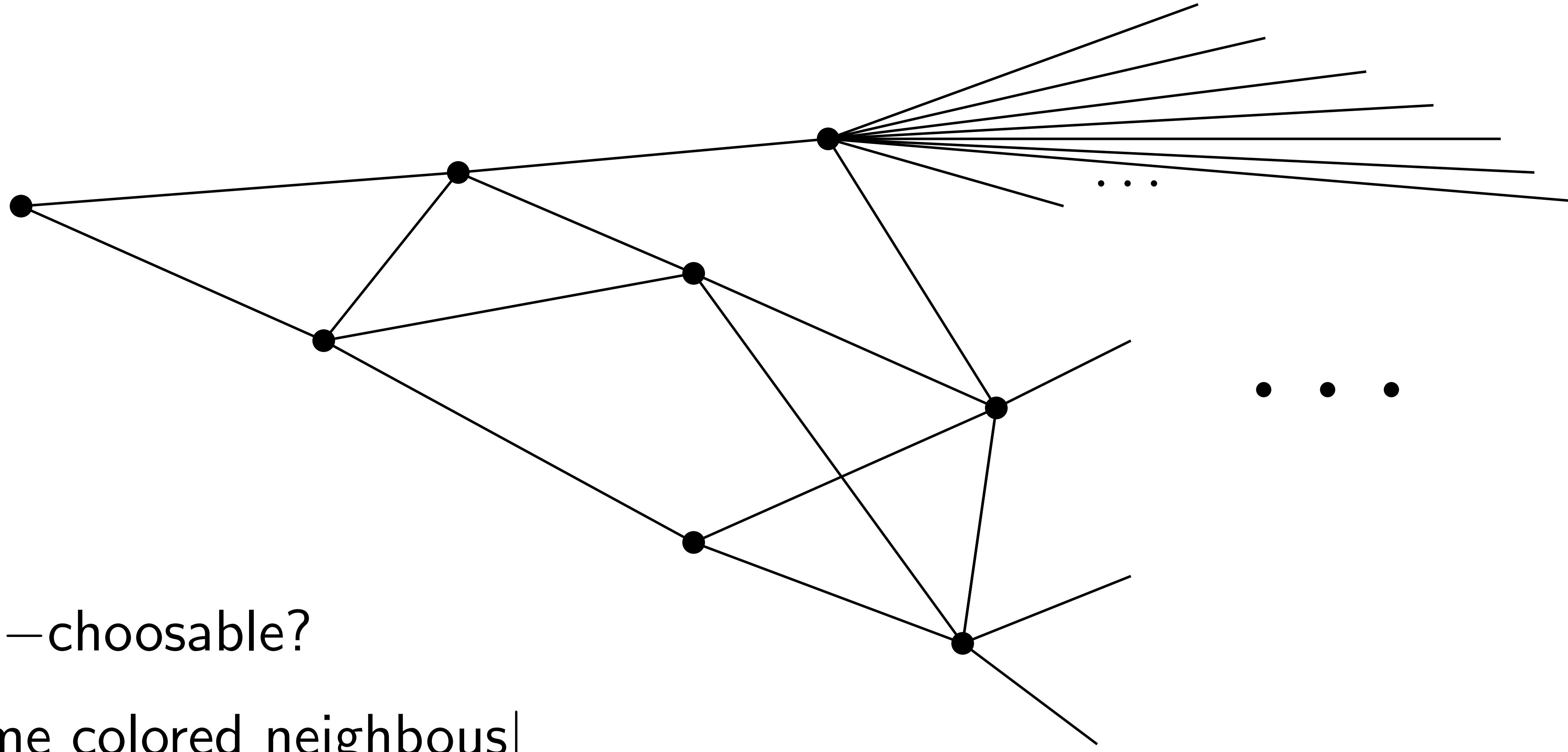

Are countable graphs majority $2-$choosable?

|Differently colored neighbous| $\geq$ |Same colored neighbous|

State of the art

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Conjecture

Every countable graph is majority $2-$colorable.

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Theorem (Haslegrave 2020)

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Overkill lemma

V - a countable set

$L : V \to \mathcal{P}(\mathbb{N})$ - a list assignment, each list has size $l + 1$

$\mathcal{X}$ - a countable family of infinite subsets of V

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Spoiler:
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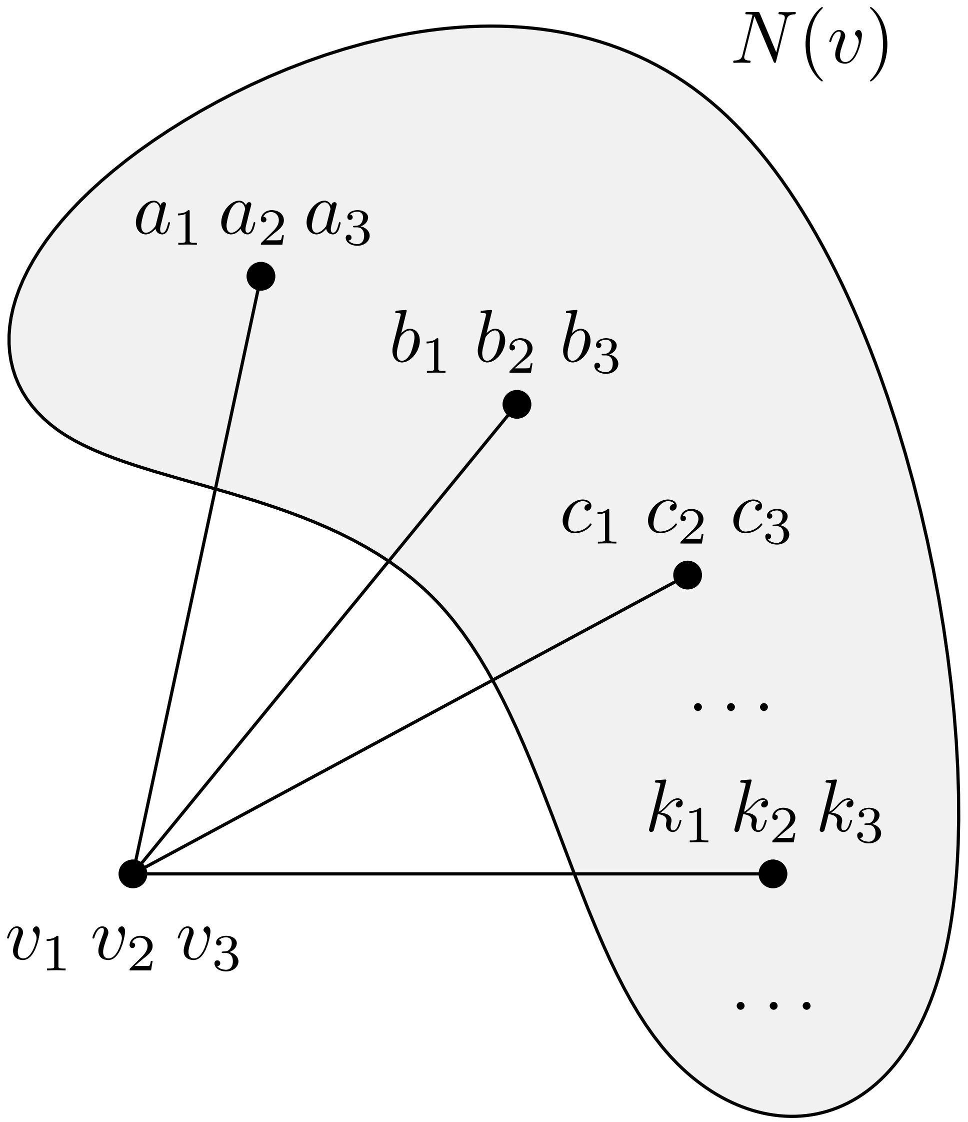

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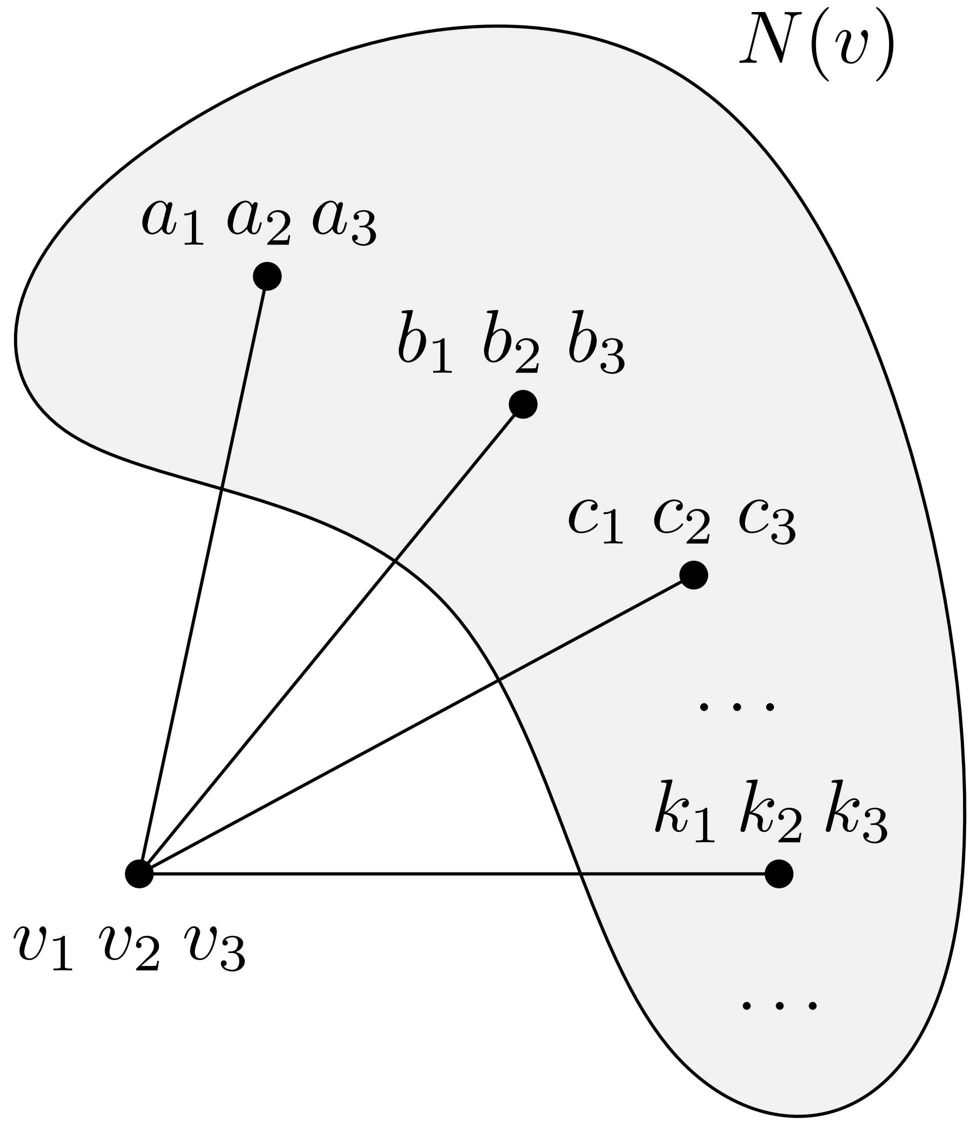

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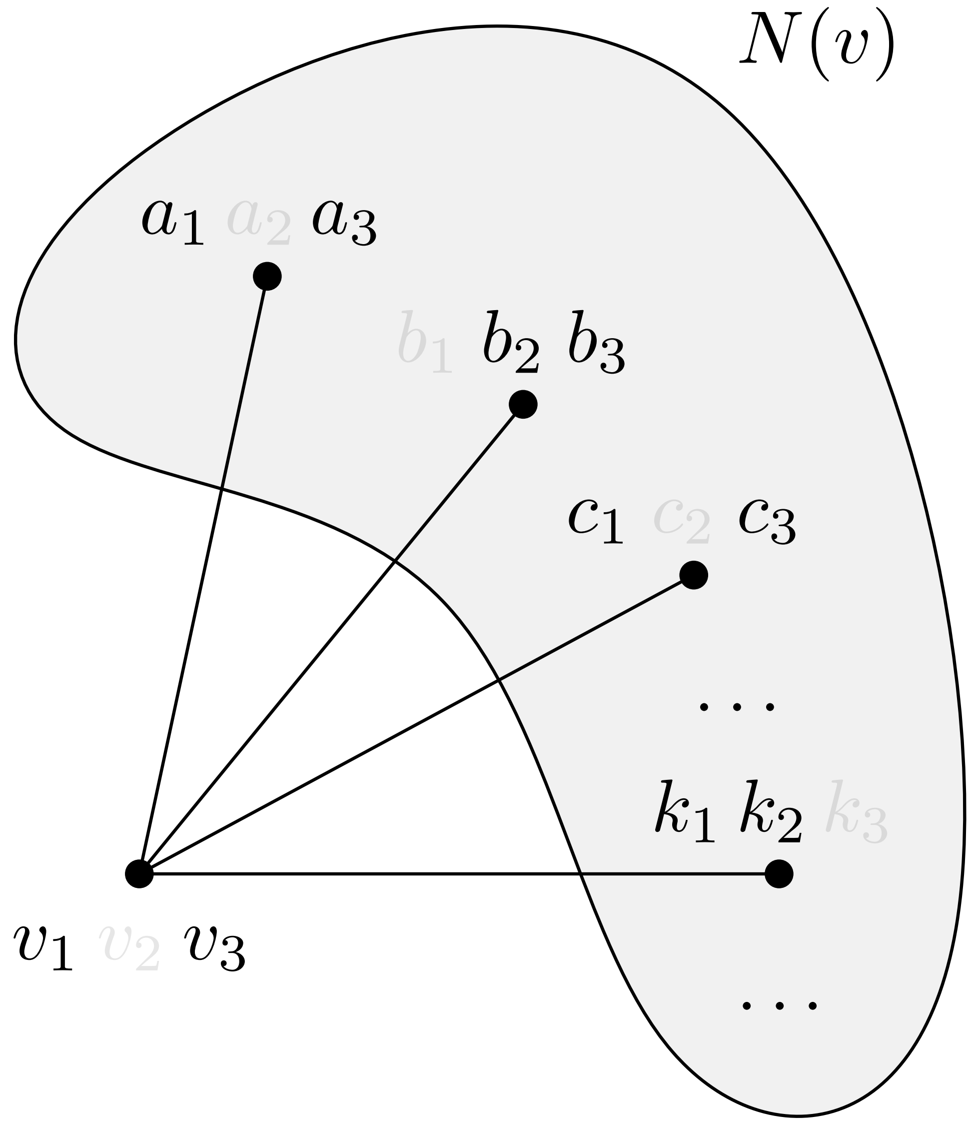

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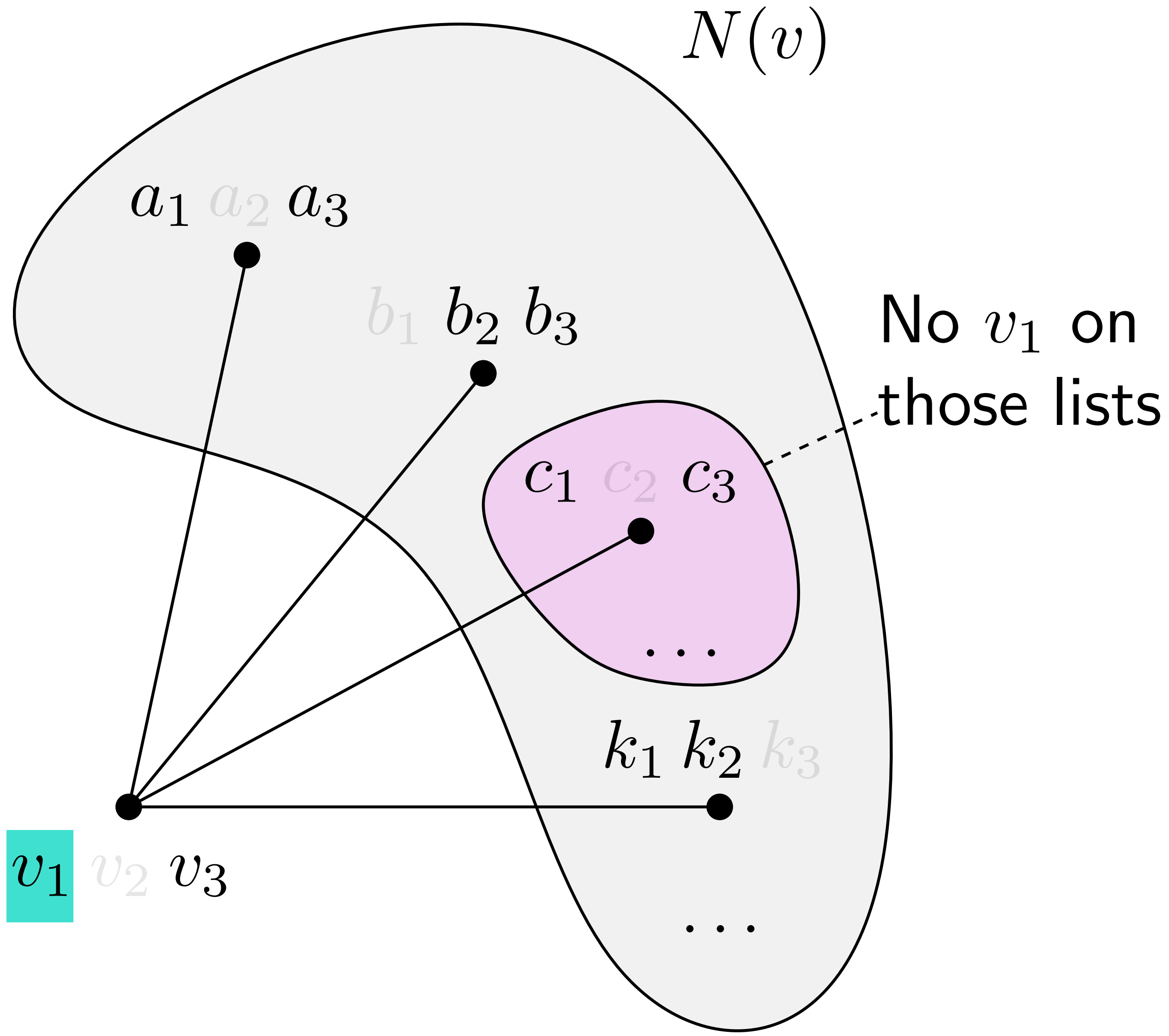

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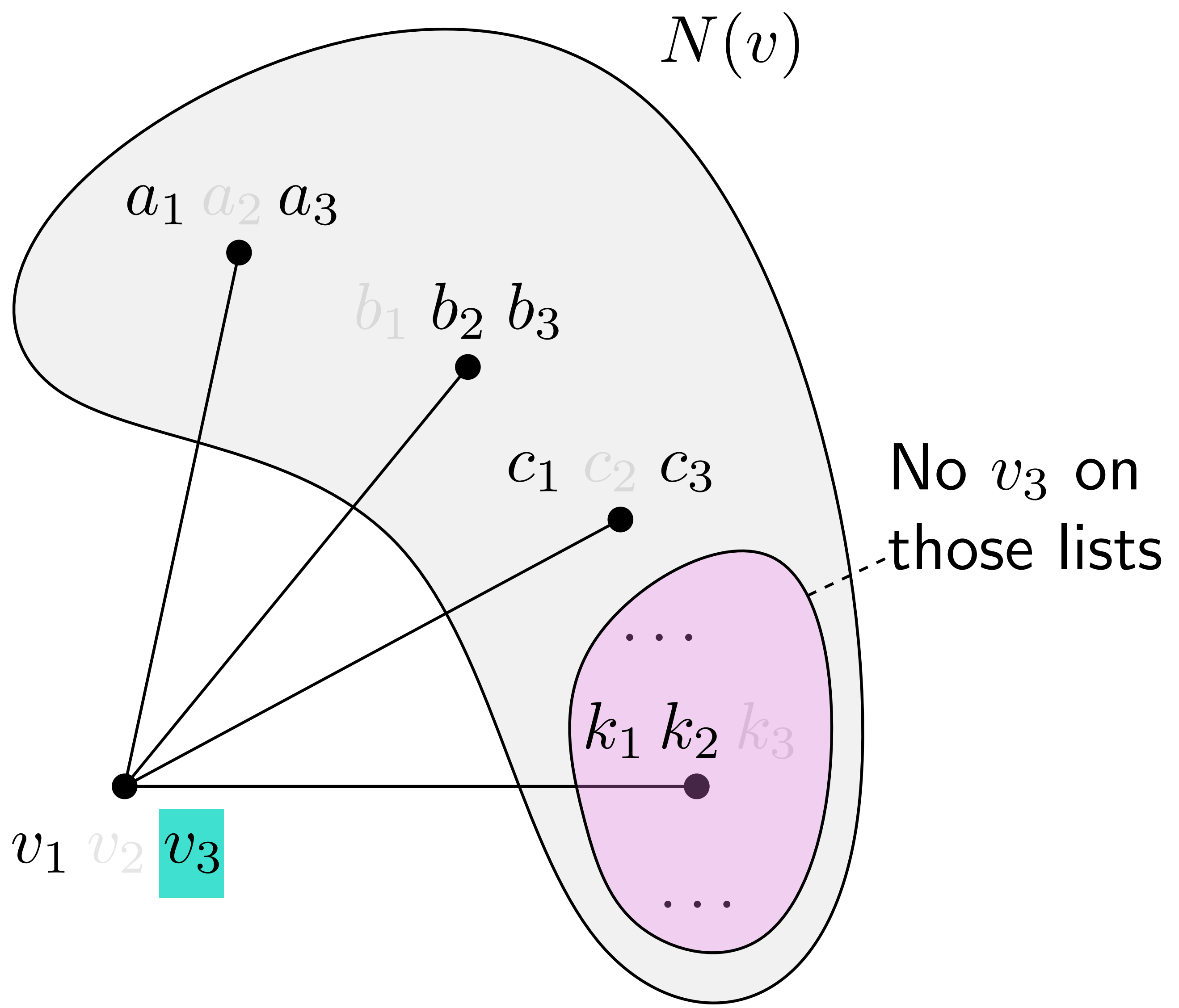

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Elem. of $\mathcal{X}$ \ Forb. color	1	2	3	$\dots$
X_1				
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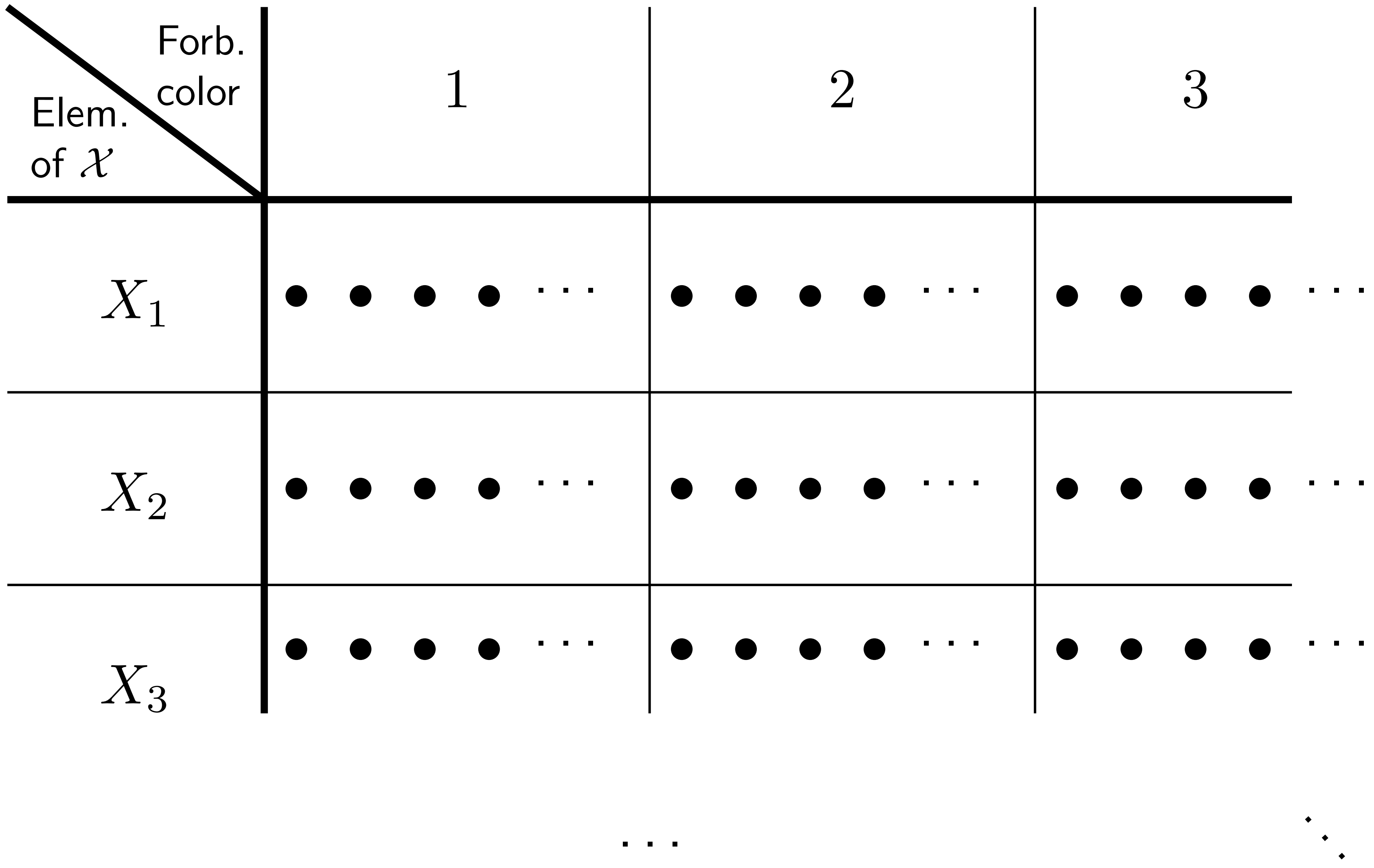

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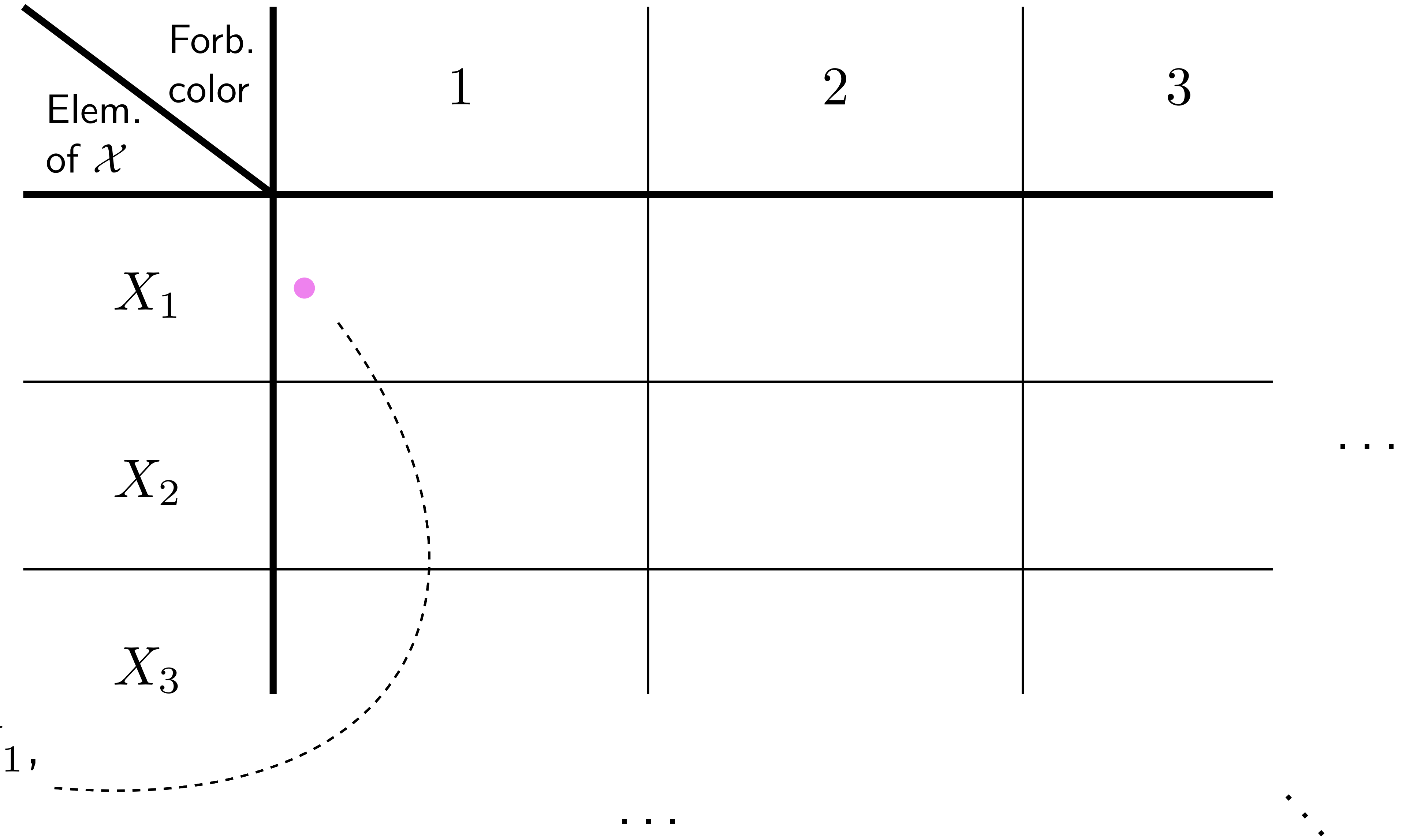

Take anything from X_1,
remove 1 from its list.

Overkill lemma

V - a countable set

$L : V \to \mathcal{P}(\mathbb{N})$ - a list assignment, each list has size $l+1$

$\mathcal{X}$ - a countable family of infinite subsets of V

Spoiler:
$l = 2$
$\mathcal{X} = \{N(v) : \deg(v) = \infty\}$

Lemma

We can select L' such that:

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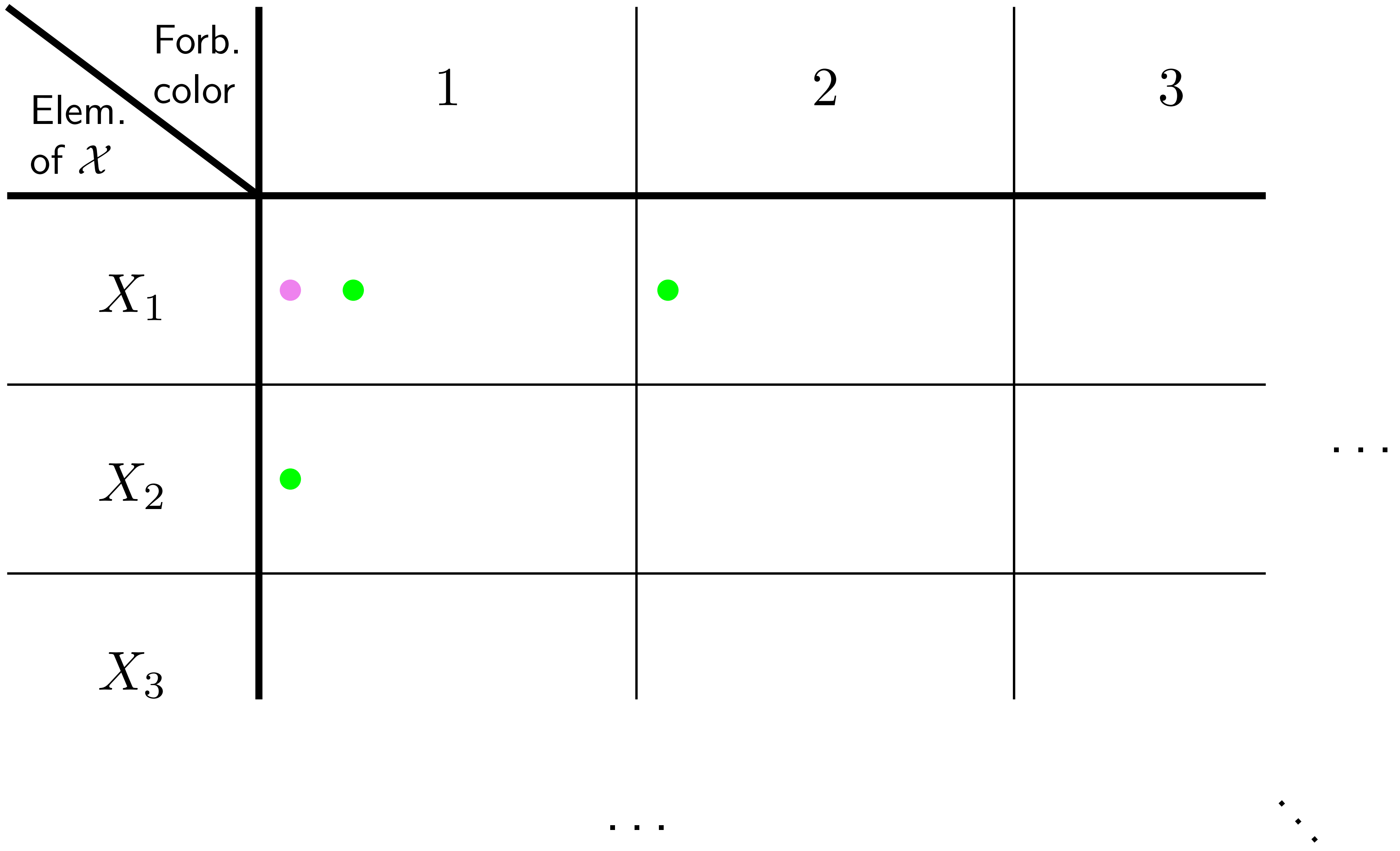

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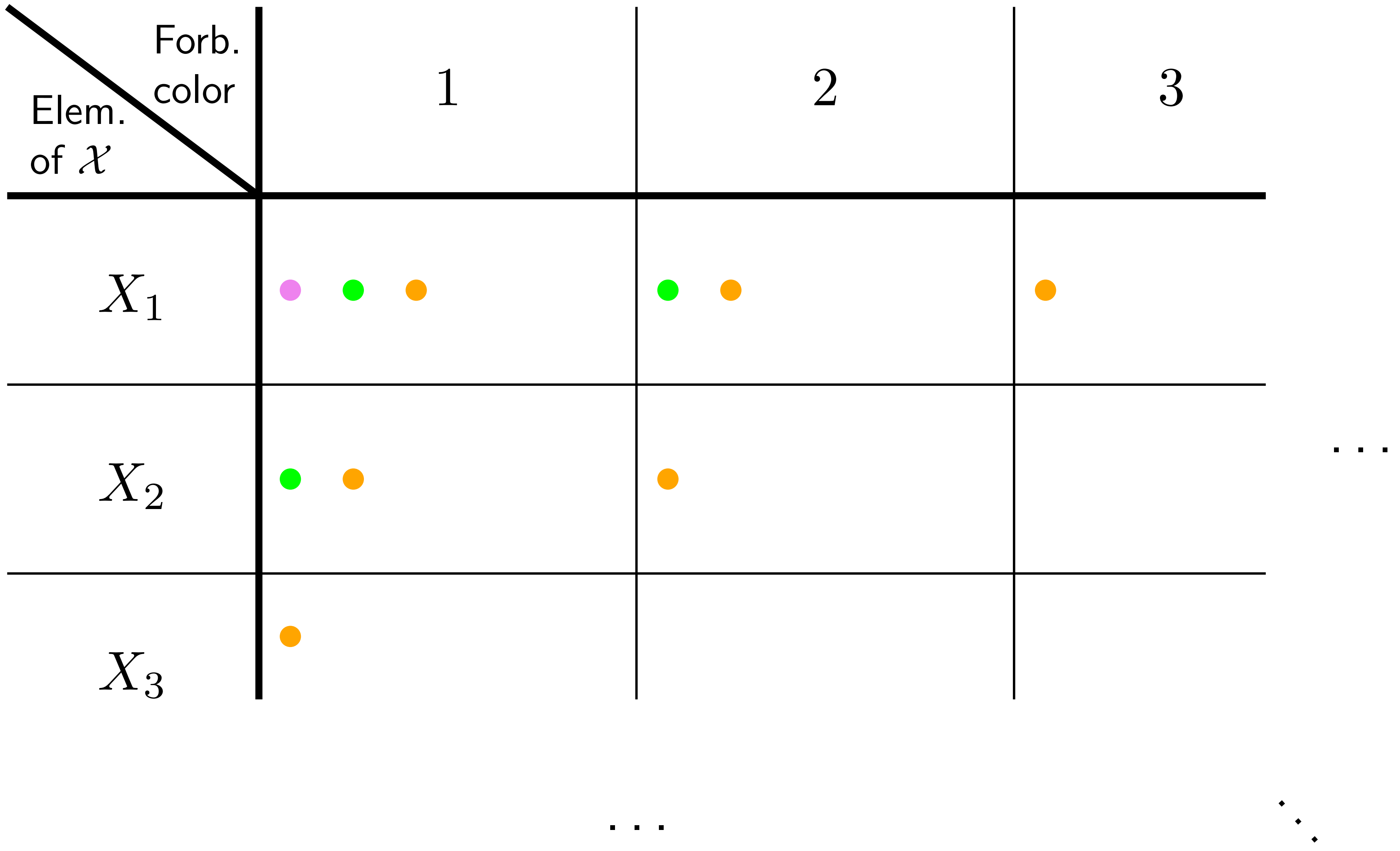

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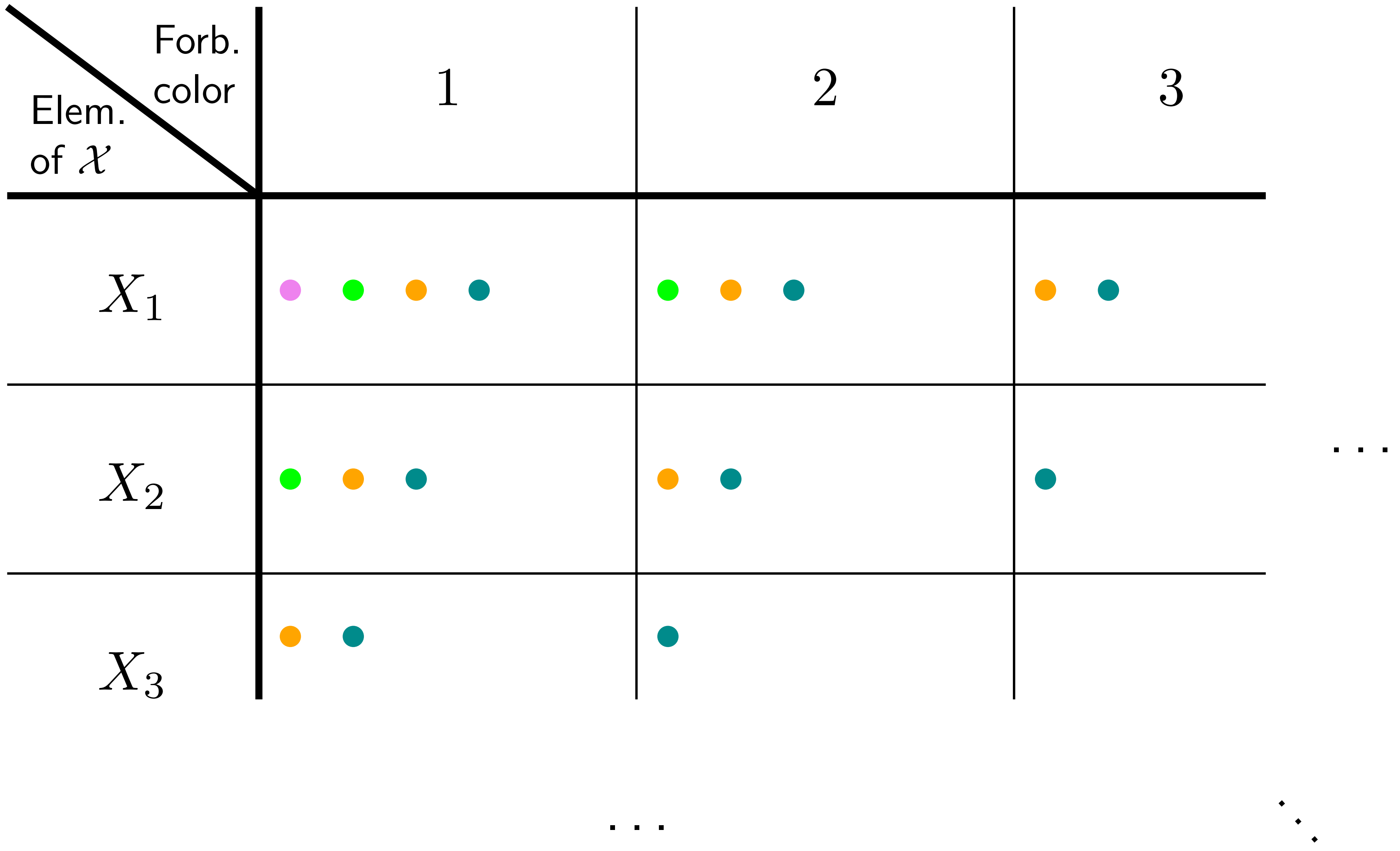

Every countable graph is majority $3-$choosable

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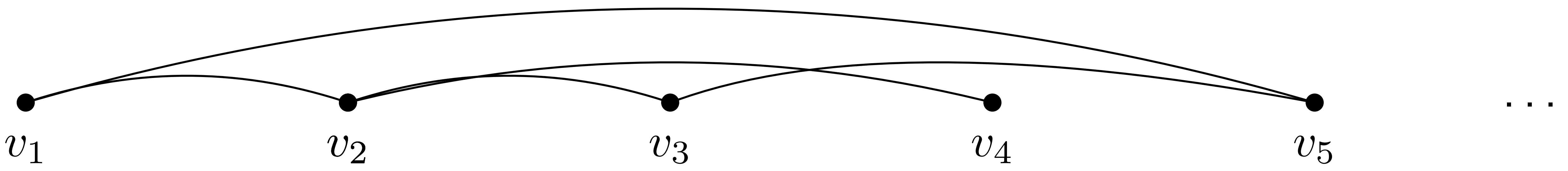

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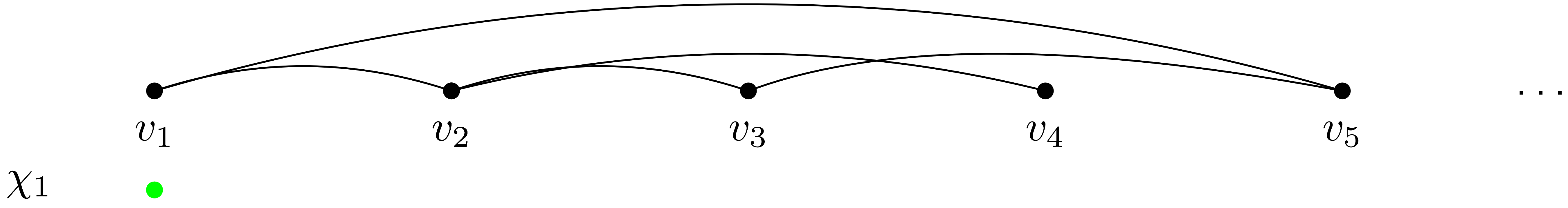

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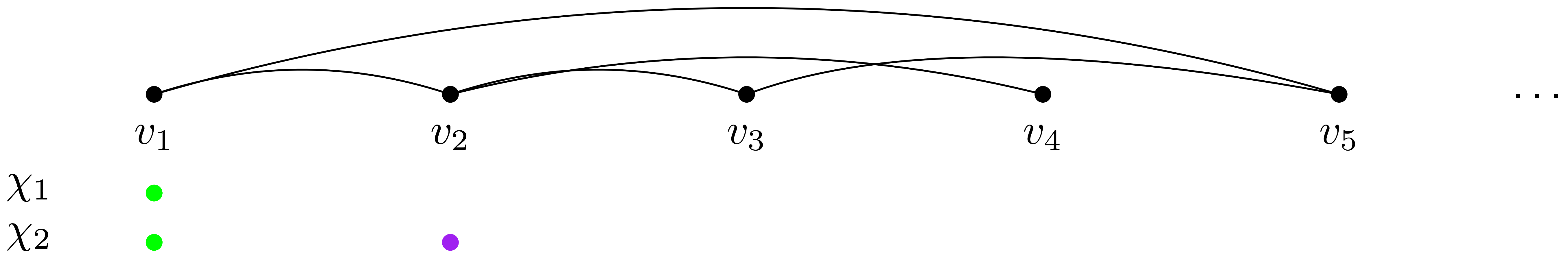

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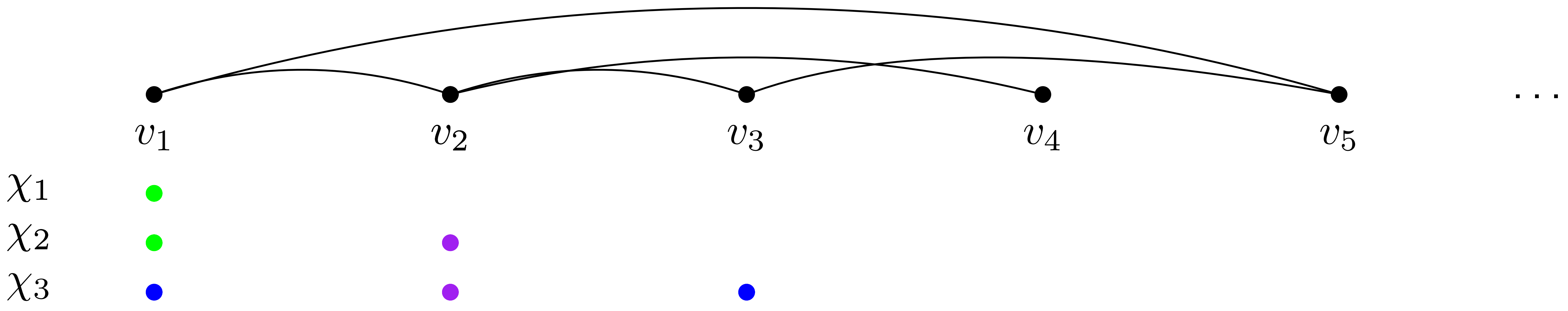

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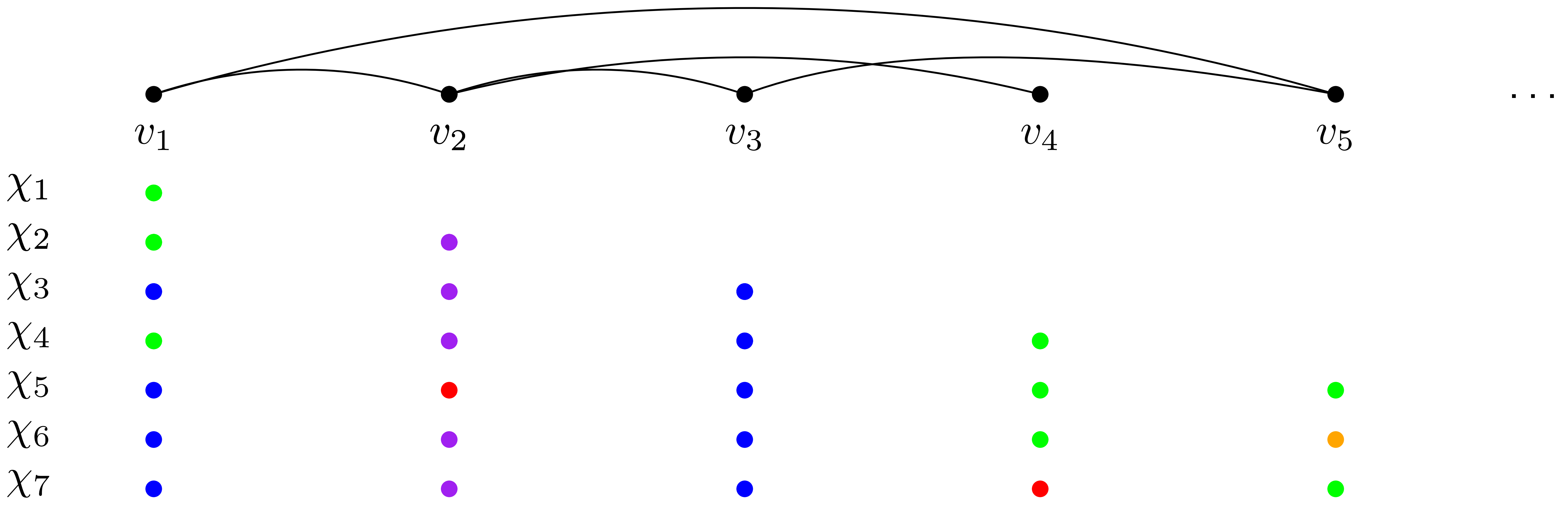

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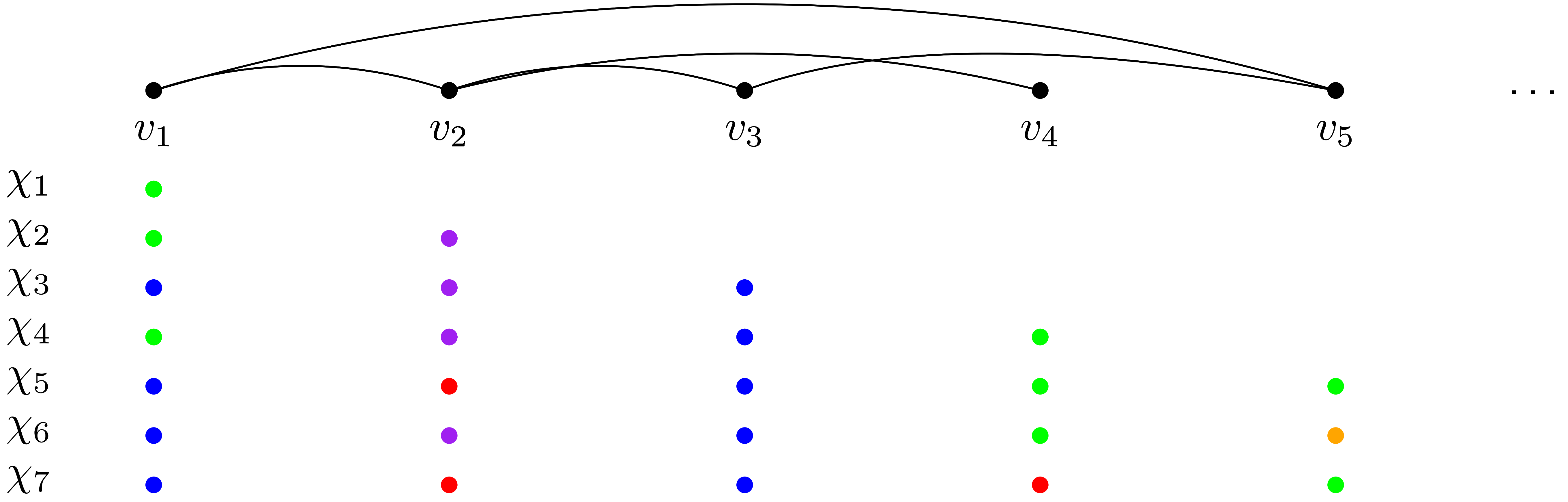

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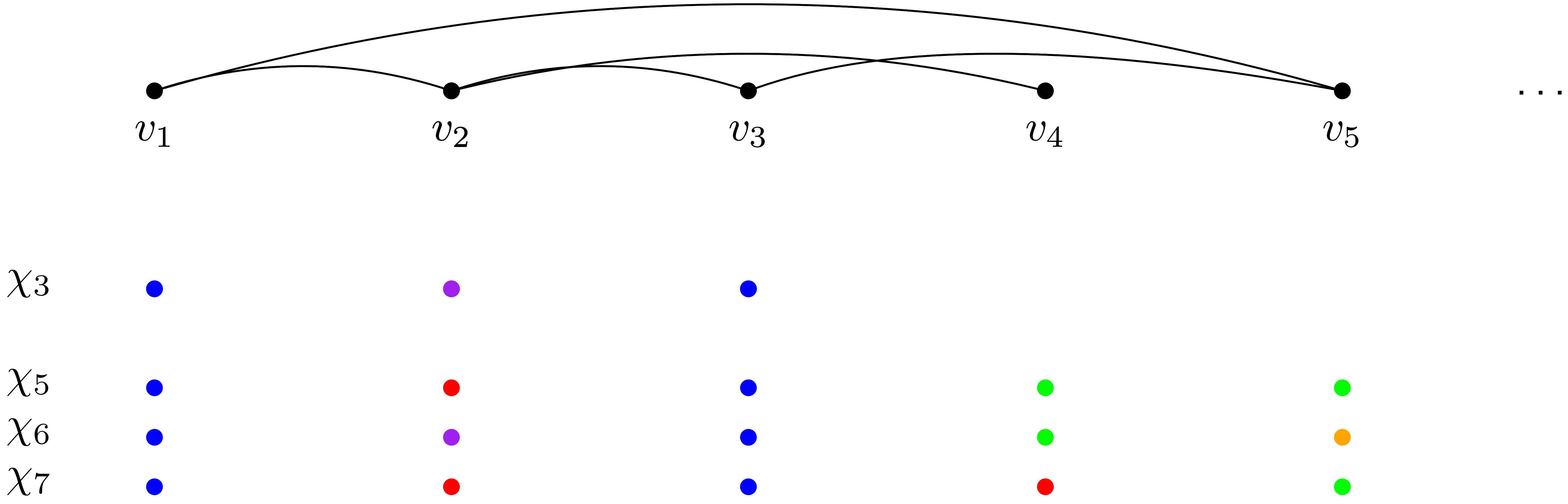

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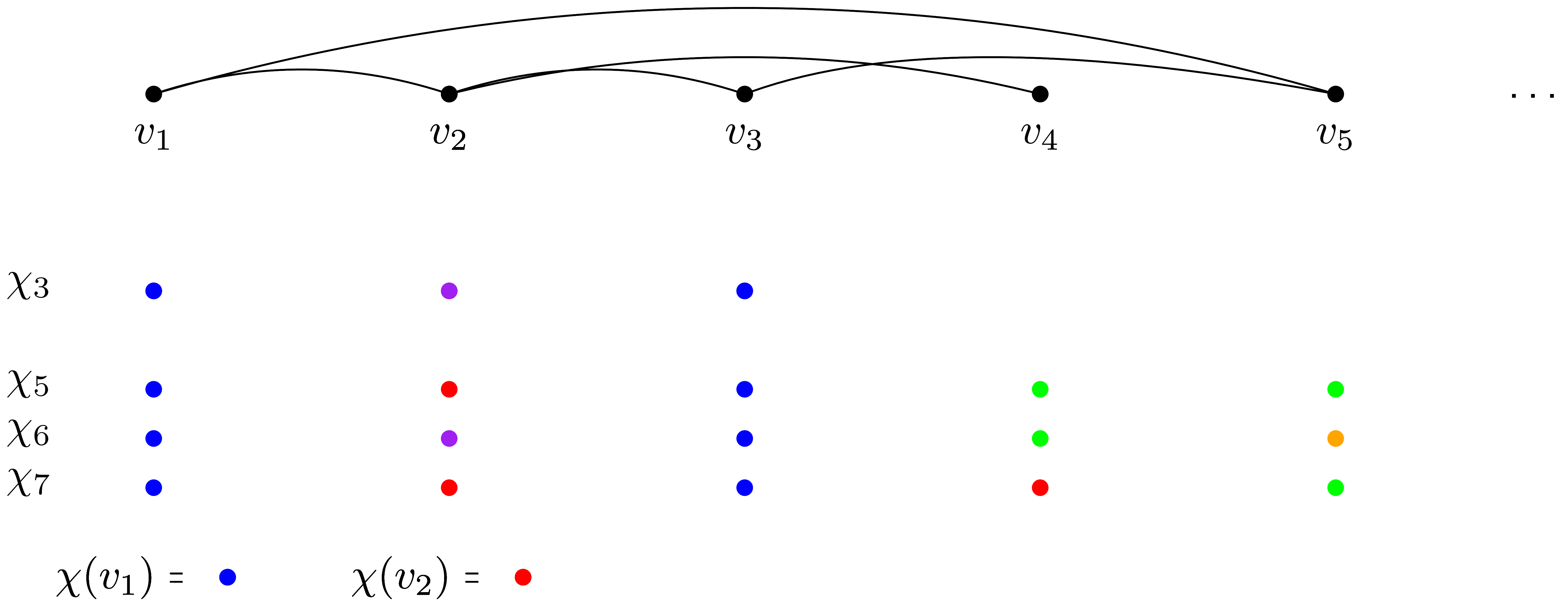

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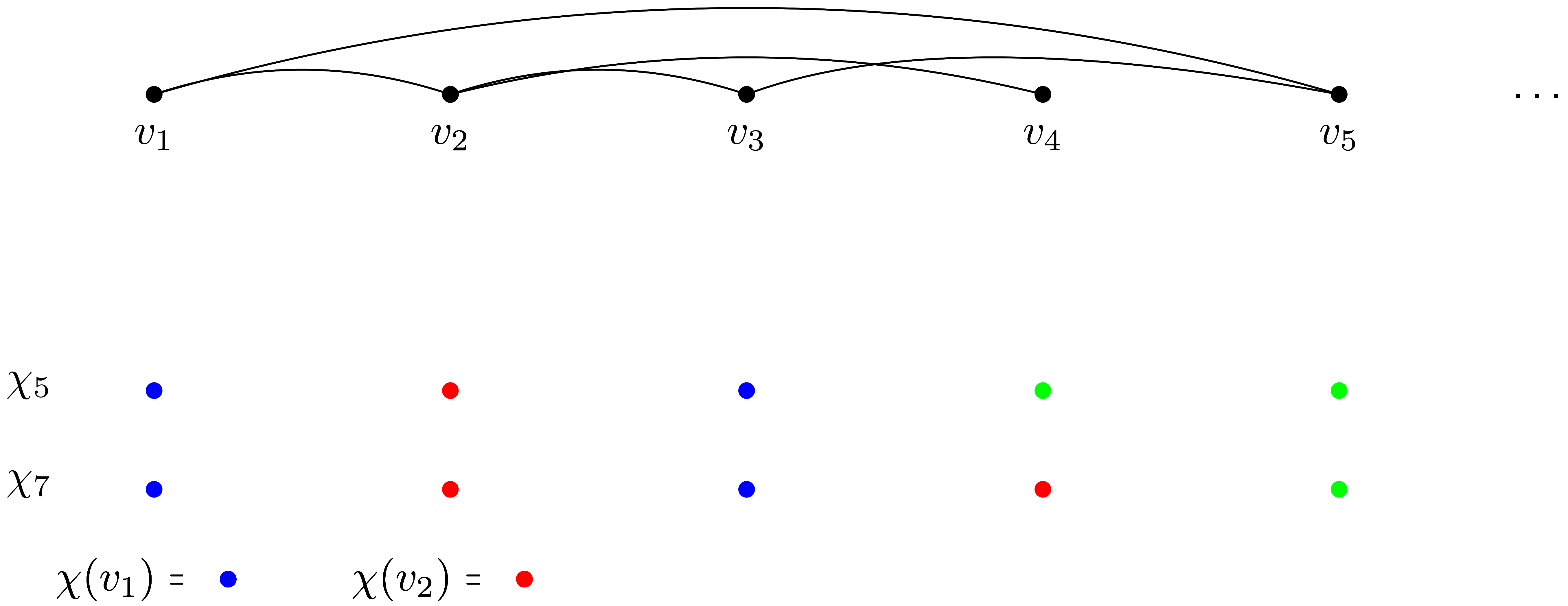

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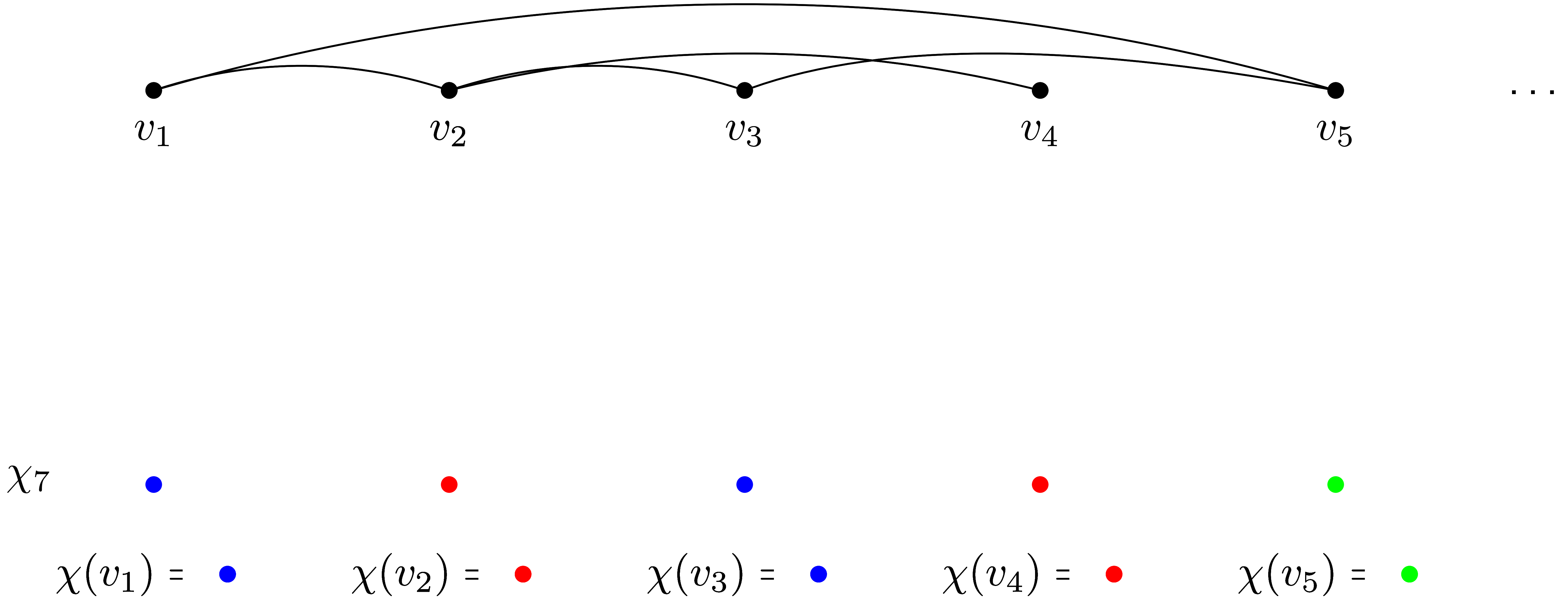

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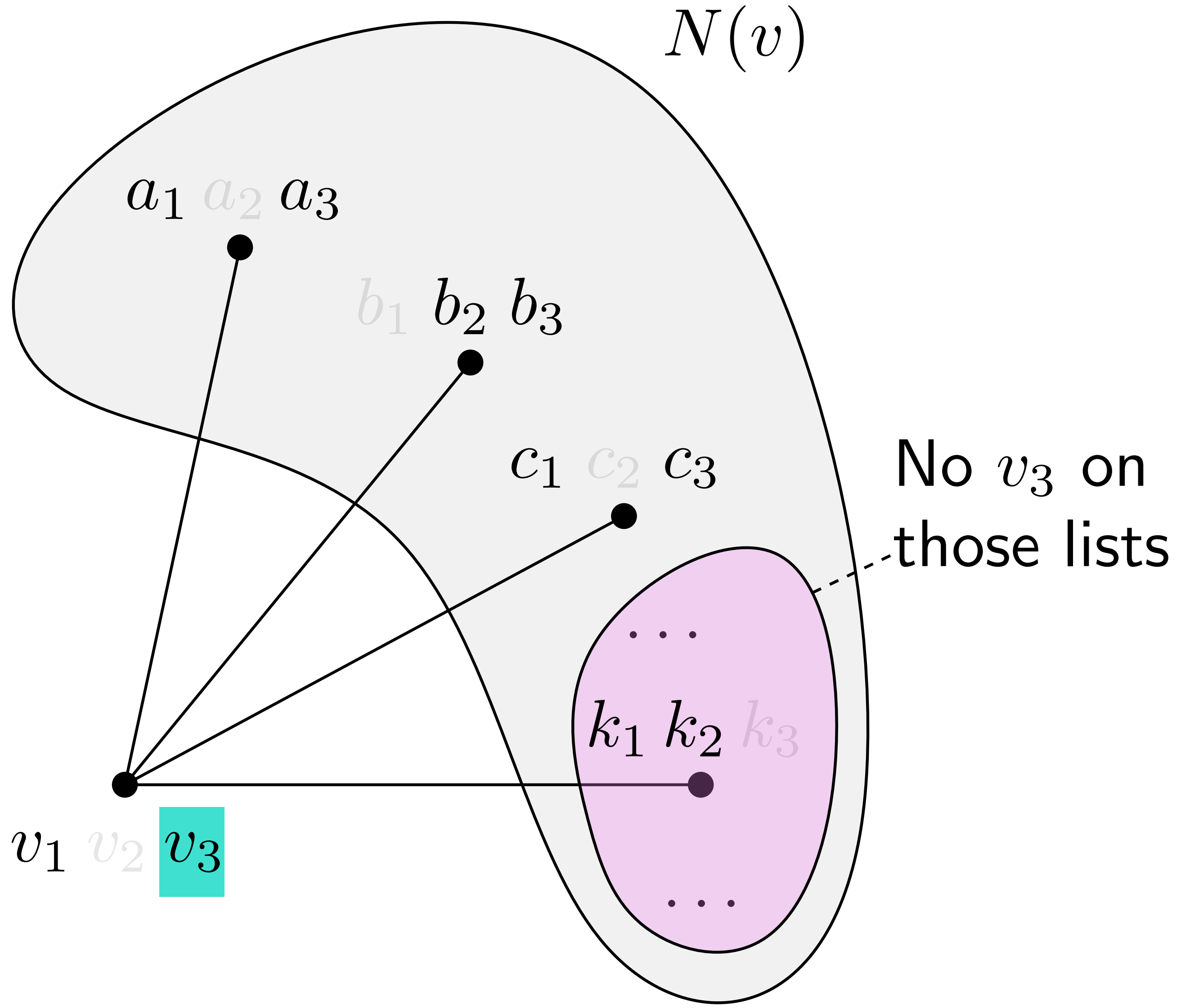

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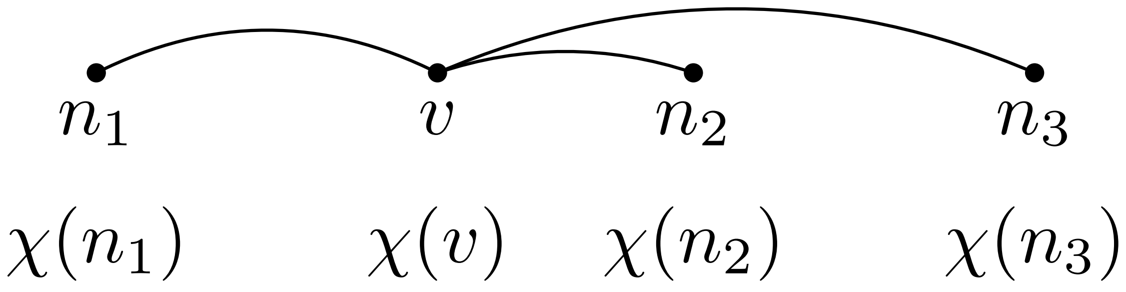

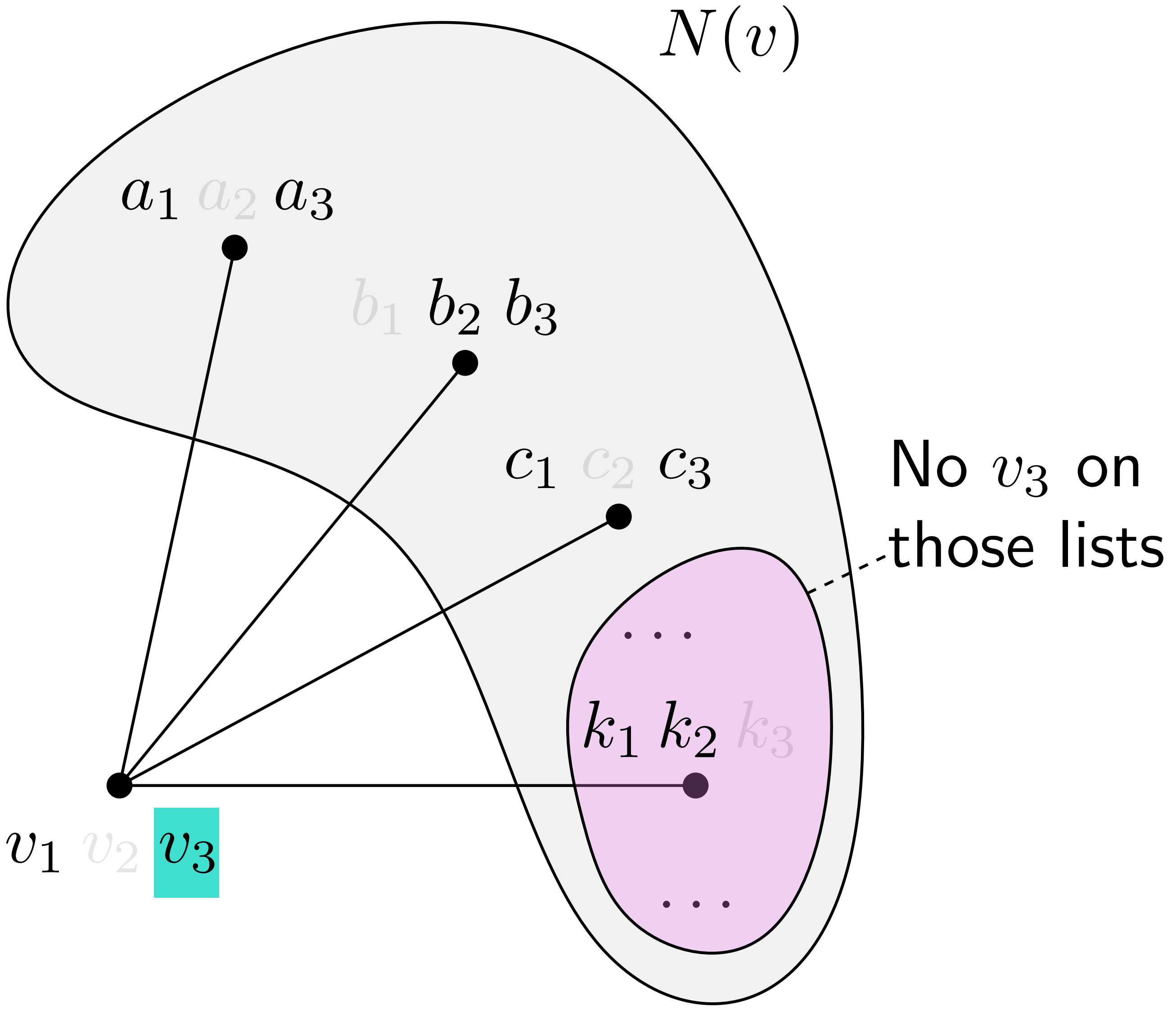

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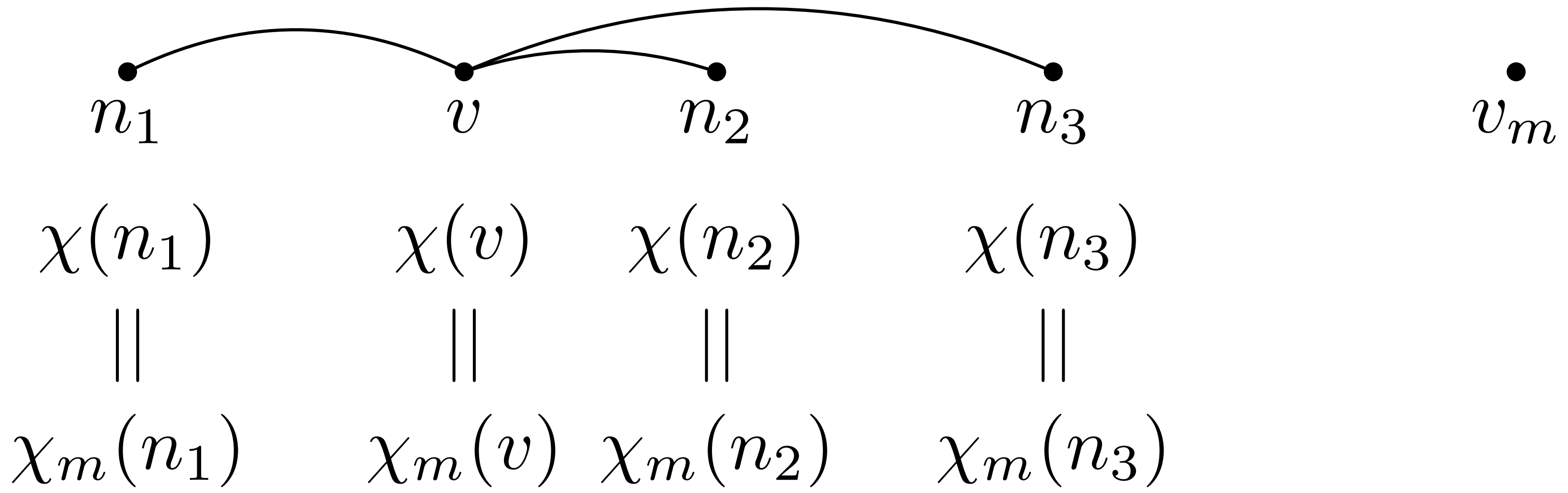

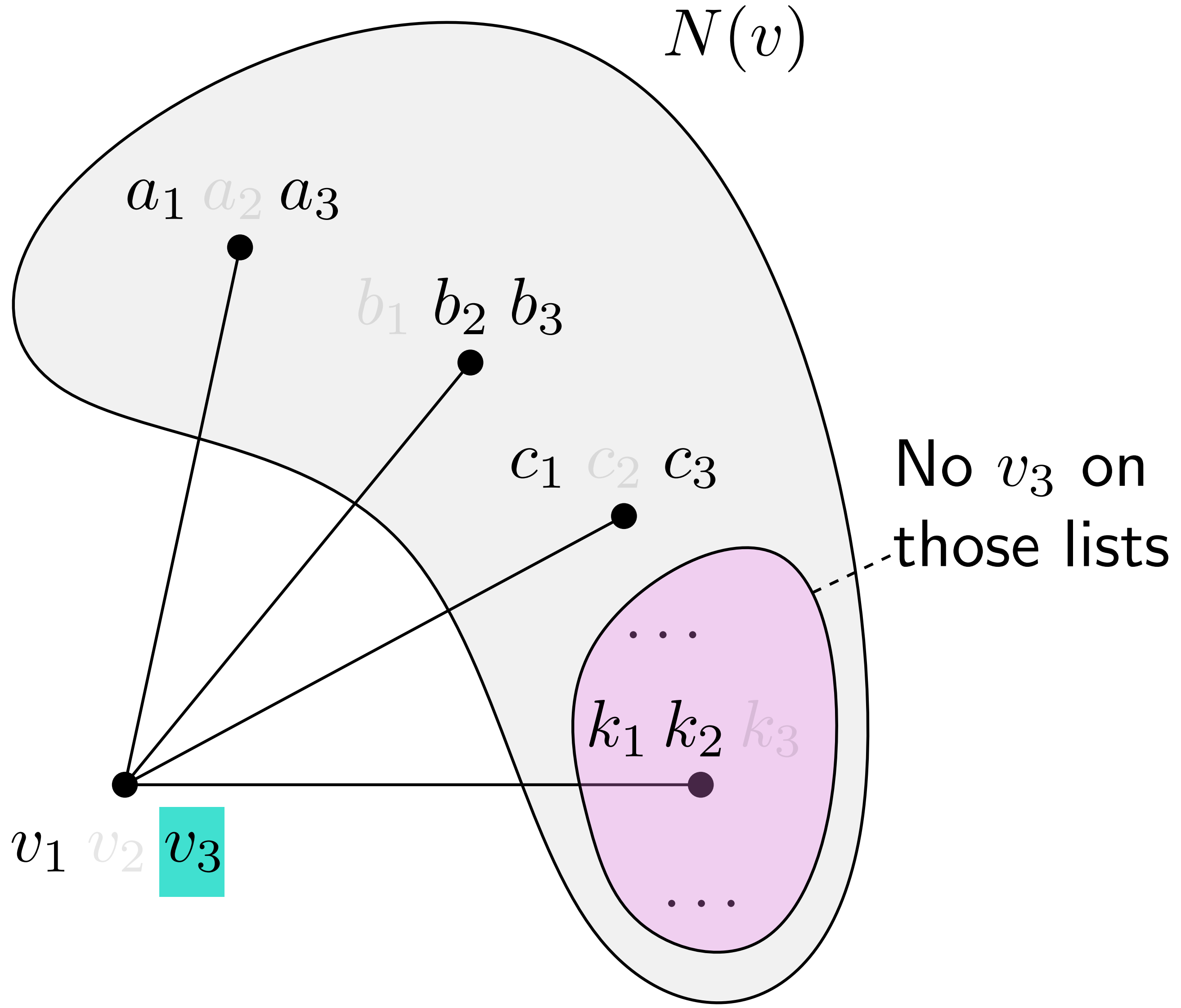

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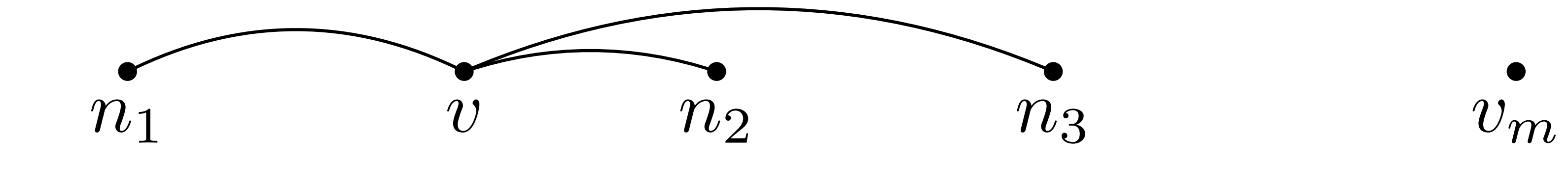

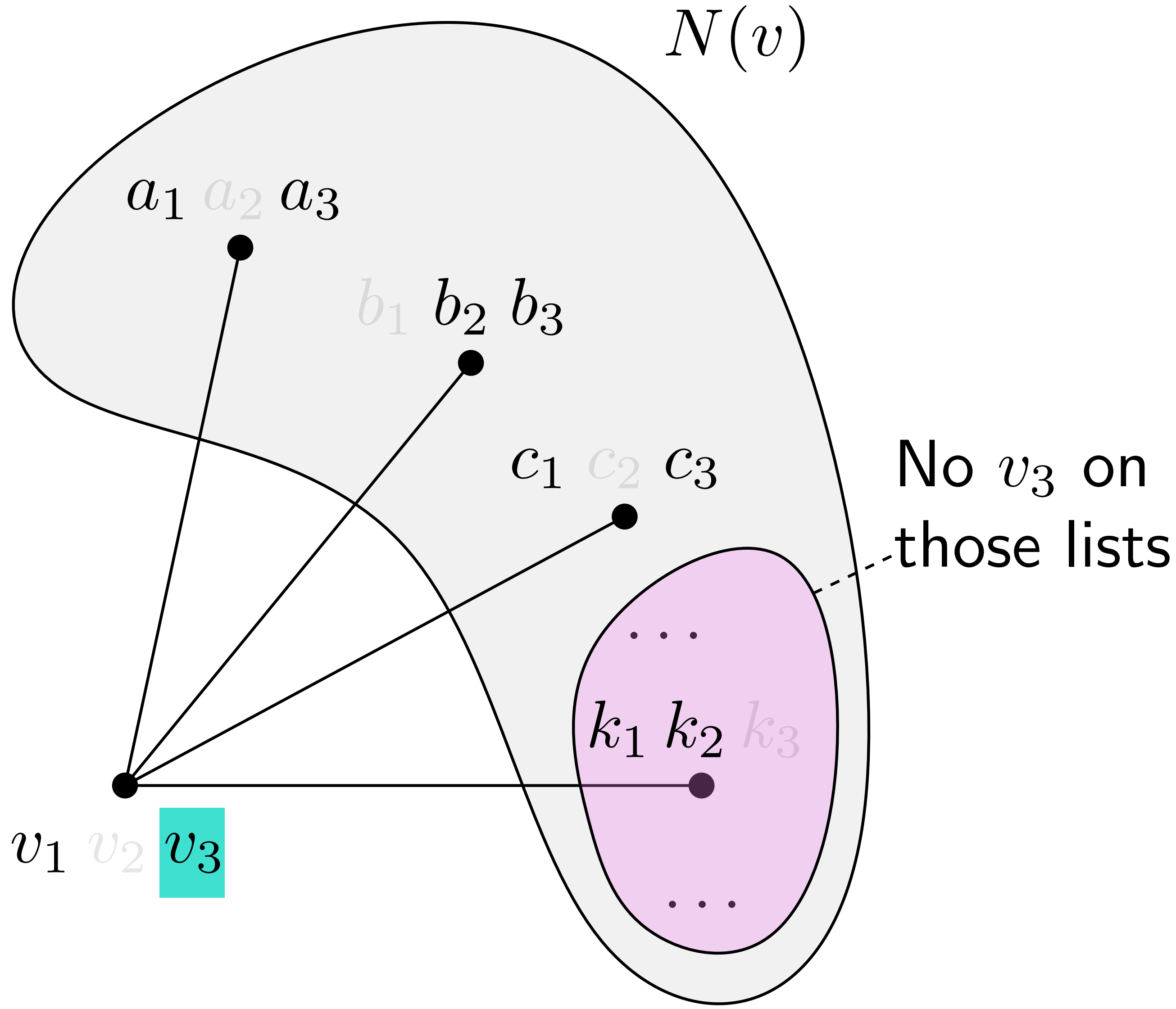

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Theorem (Haslegrave 2020)

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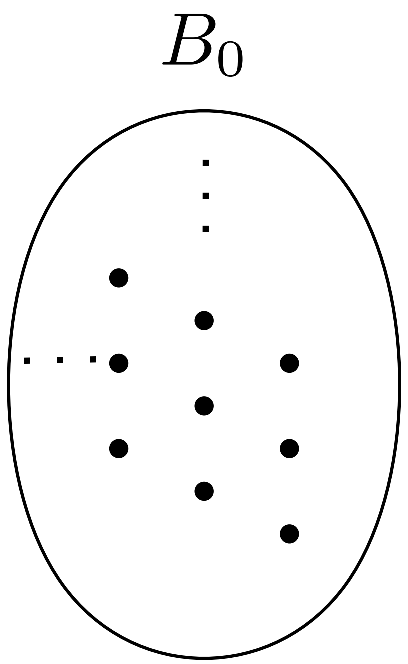

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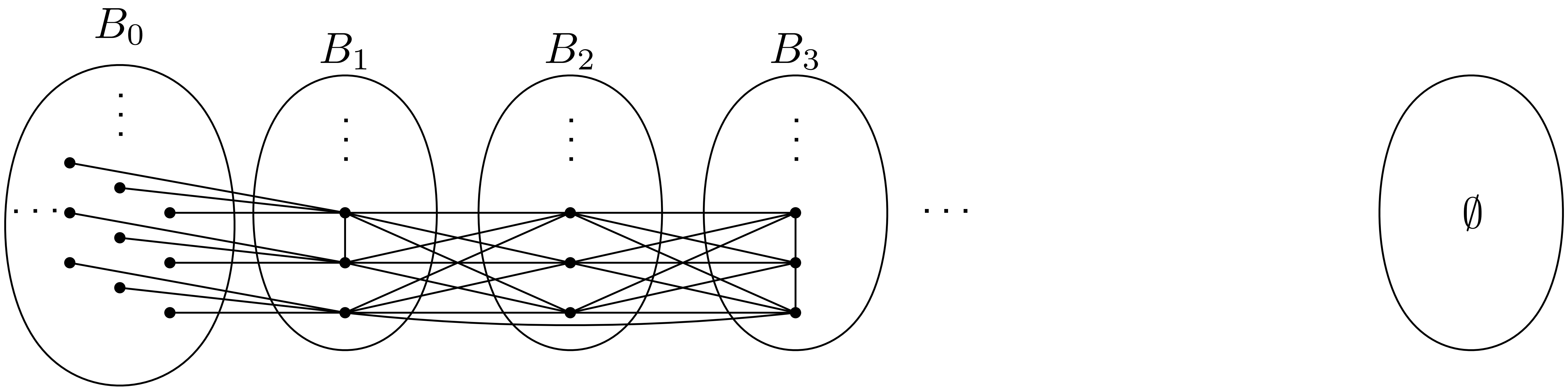

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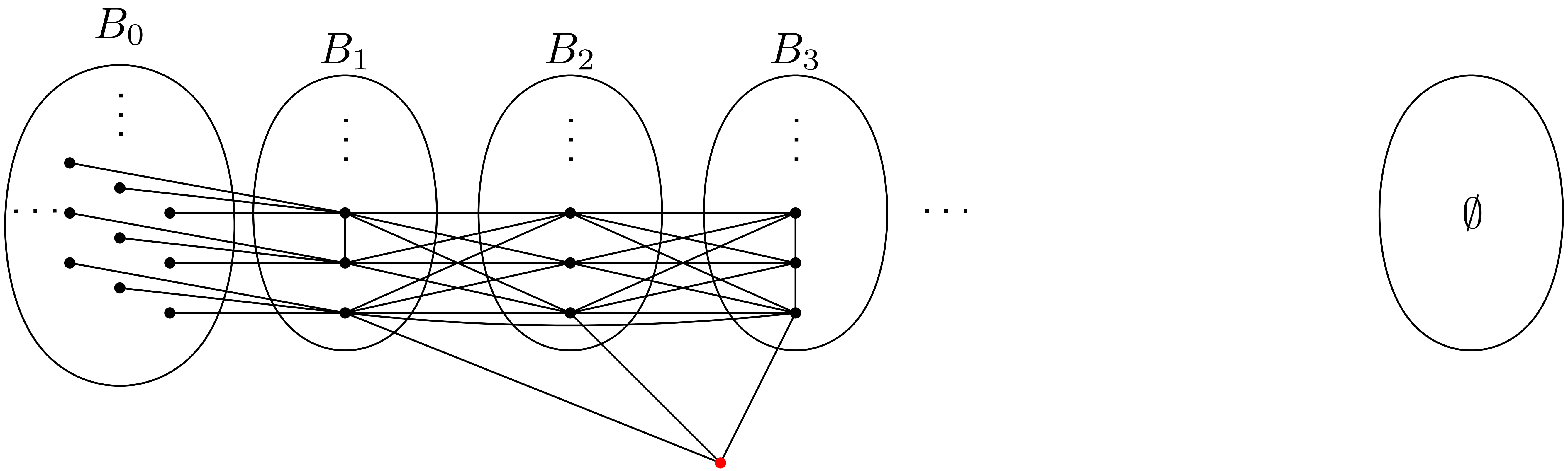

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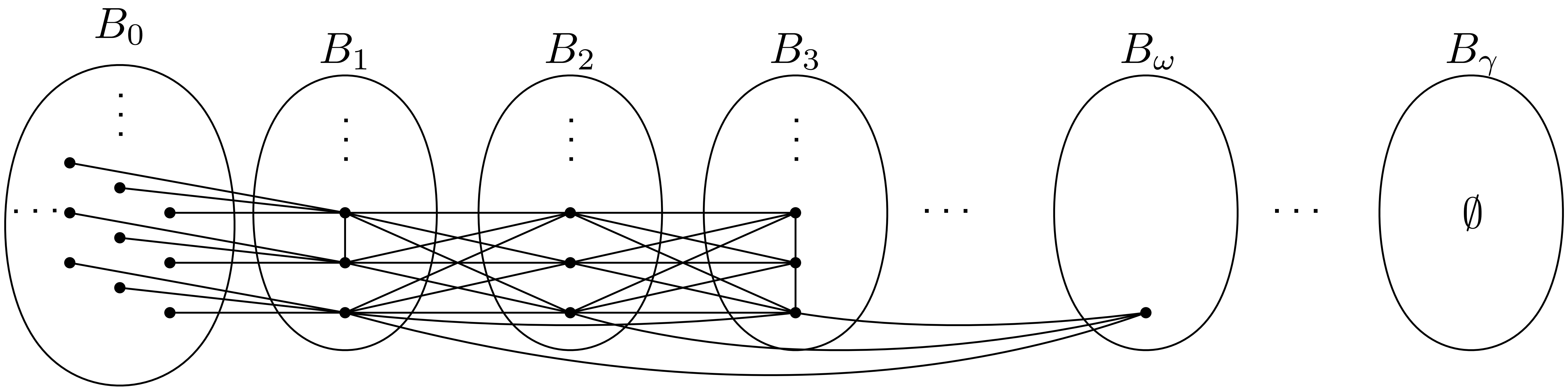

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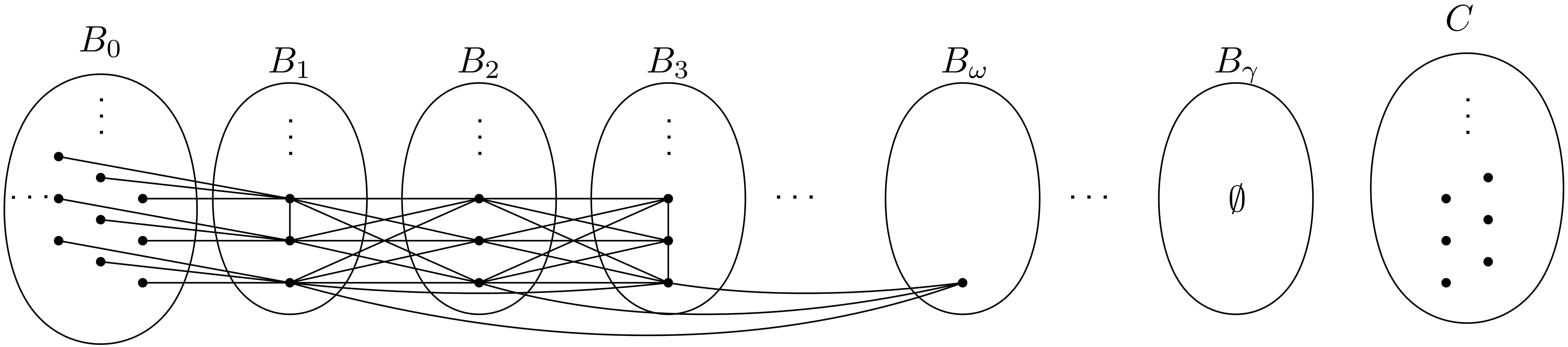

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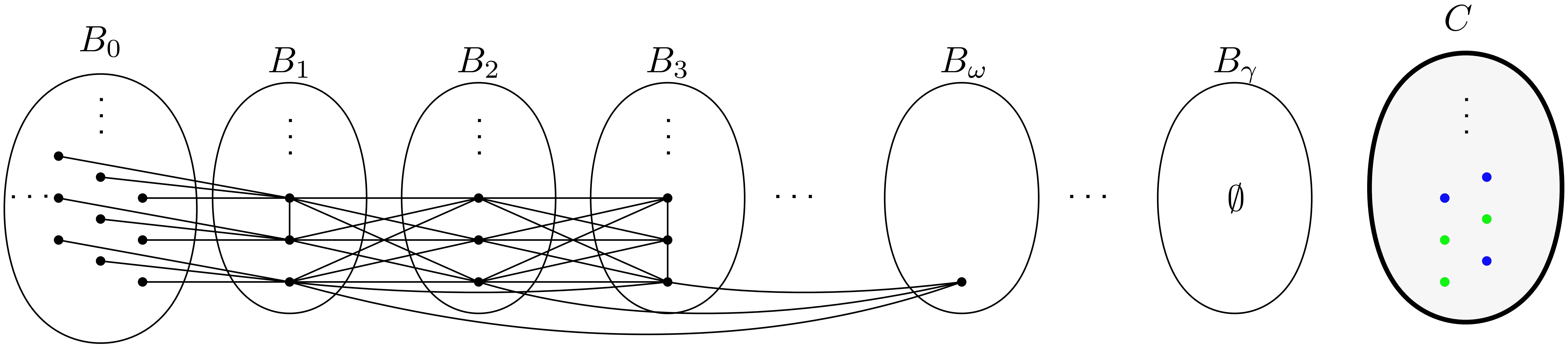
(Implies that) every graph with finitely many vertices of finite degree is majority $2-$colorable.



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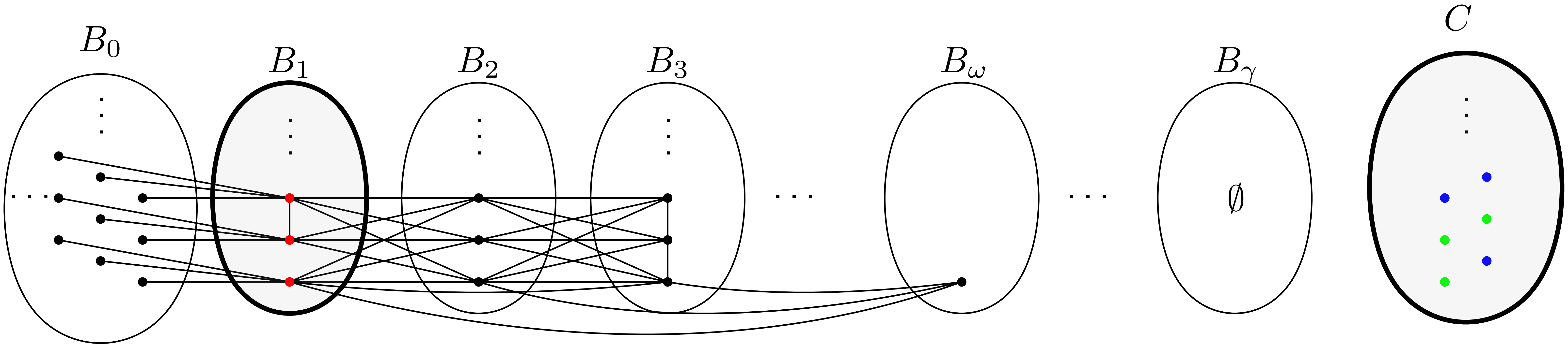

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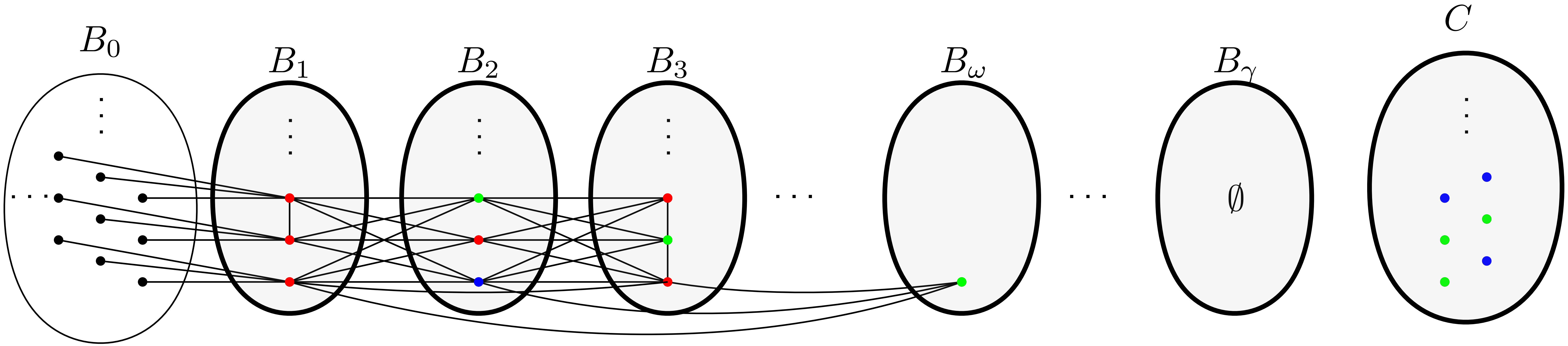

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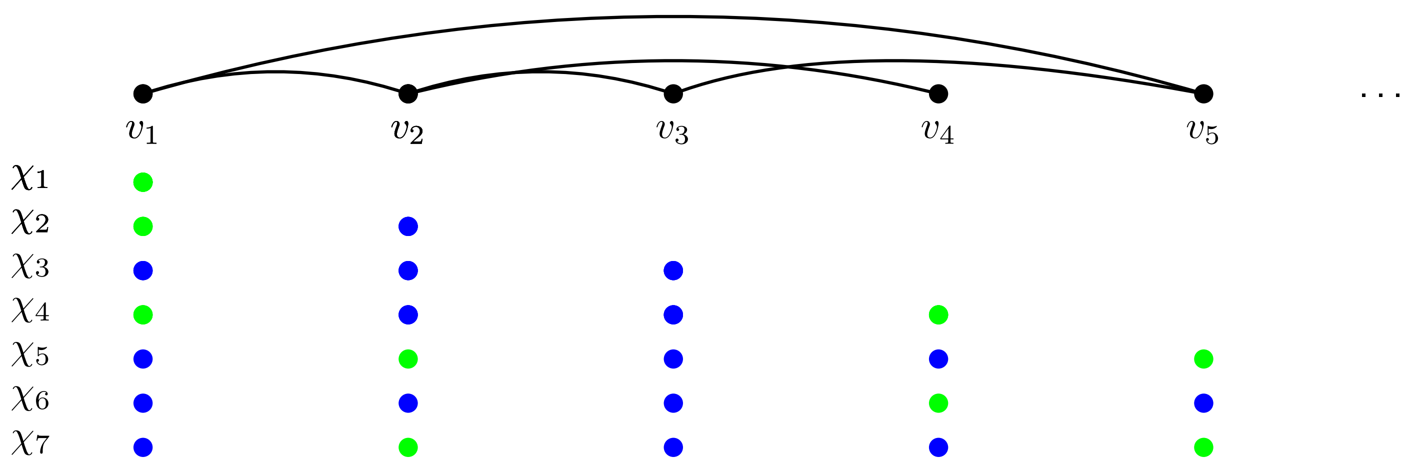

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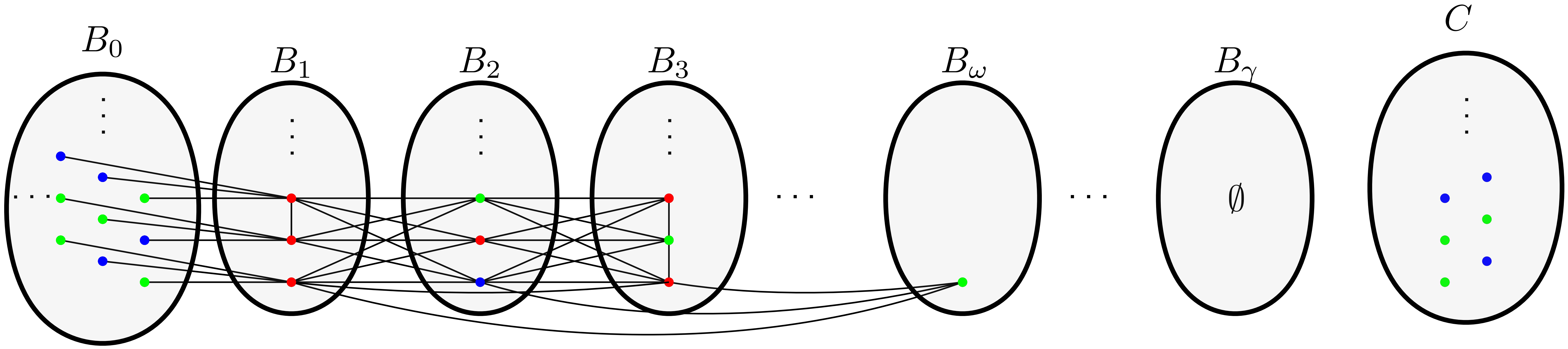

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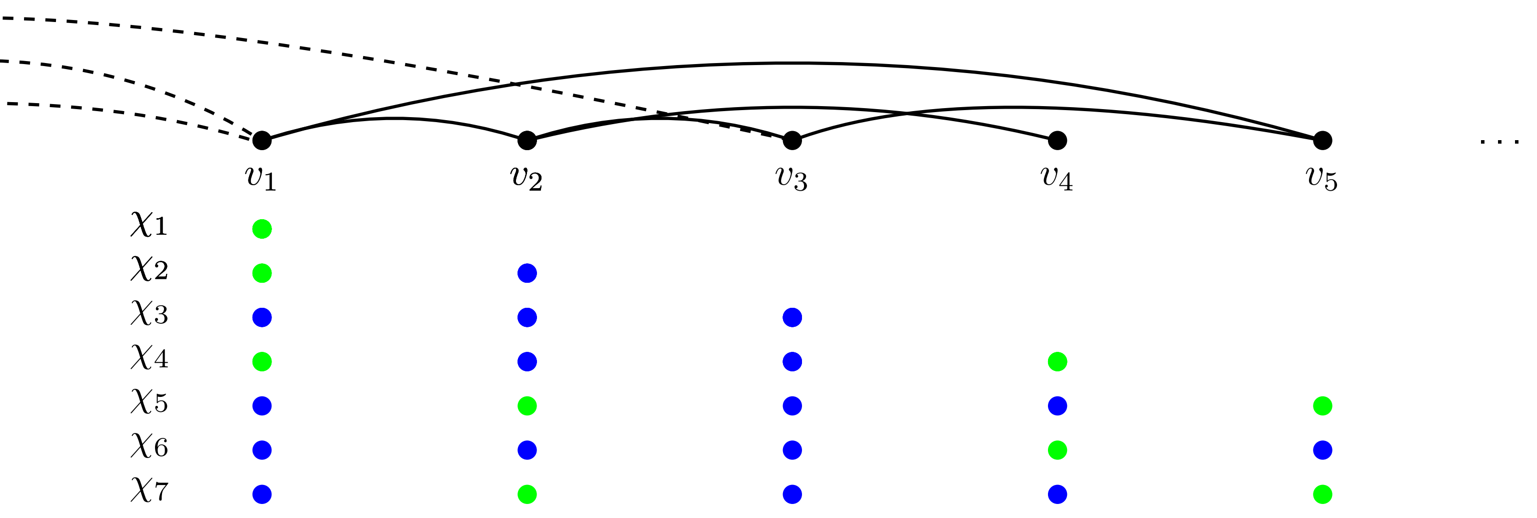

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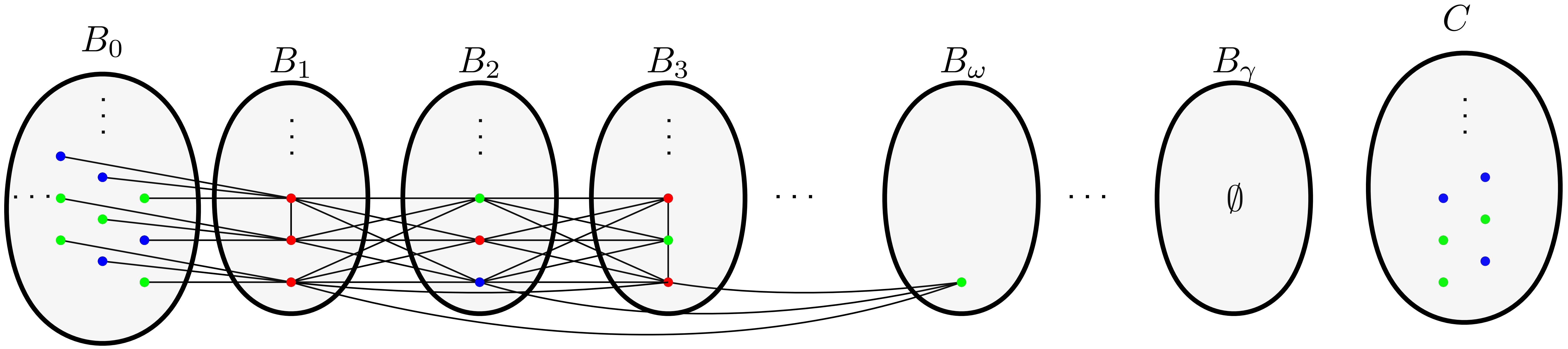

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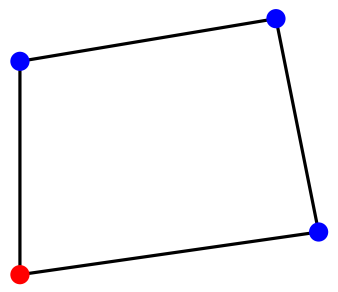
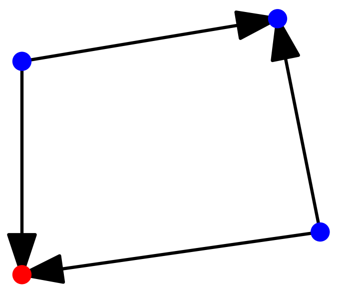
Theorem (Haslegrave 2020)

Every countable acyclic digraph is majority $3-$choosable.





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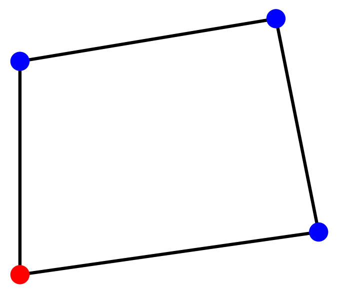

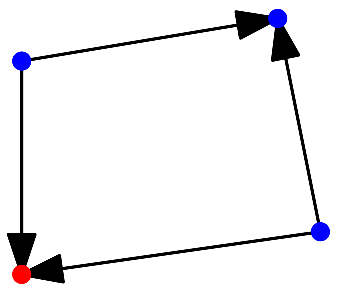

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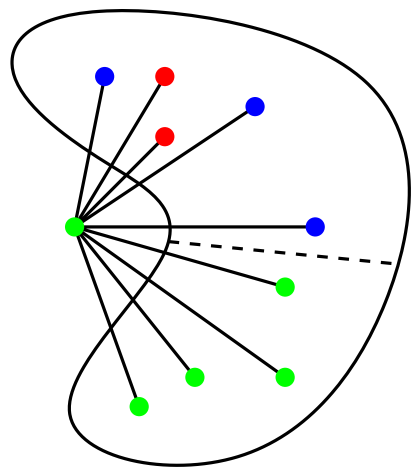
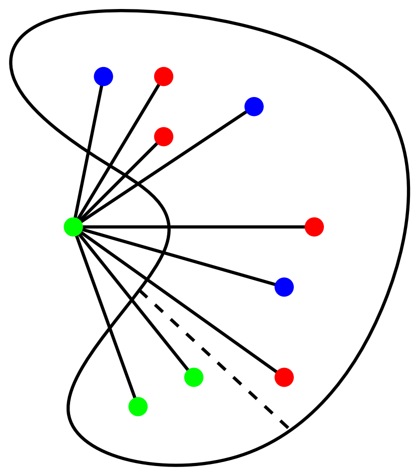


At most $1/2$



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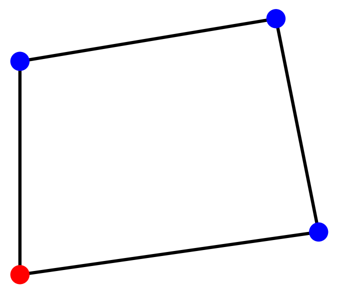

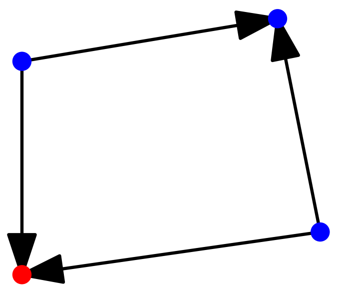

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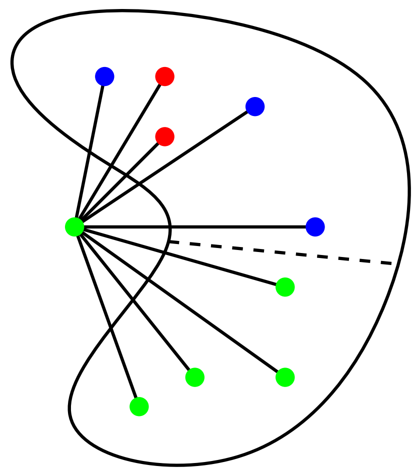
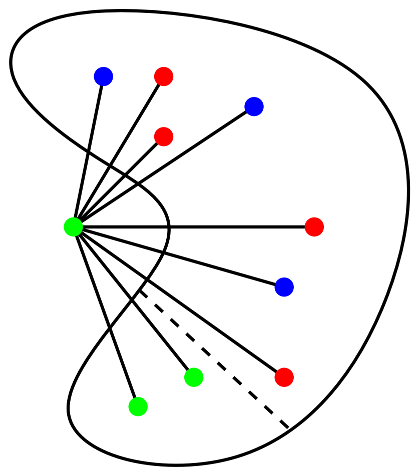


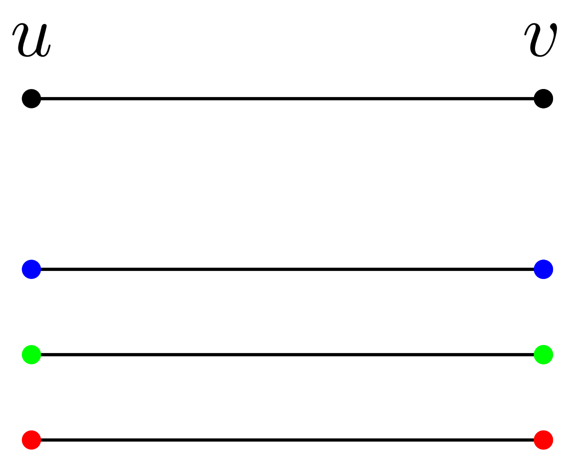
At most $1/2$

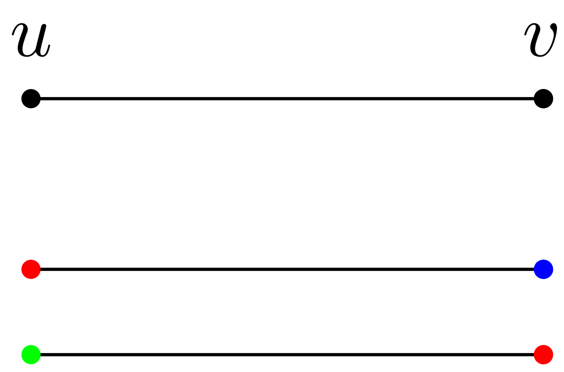


At most $1/k$

Theorem (Haslegrave 2020)

For each $k \geq 2$, any countable digraph or countable acyclic digraph is $(1/k)-$majority $(k+1)-$correspondence colorable.





Open problems

Conjecture

Every countable graph is majority $2-$colorable ($2-$choosable).

Conjecture

Every (countable) digraph is majority $3-$colorable ($3-$choosable).

Conjecture

Every countable acyclic digraph is majority $2-$colorable ($2-$choosable).

Open problems

Conjecture

Every countable graph is majority $2-$colorable ($2-$choosable).

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Conjecture

Every countable acyclic digraph is majority $2-$colorable ($2-$choosable).

Thank you
for attention!